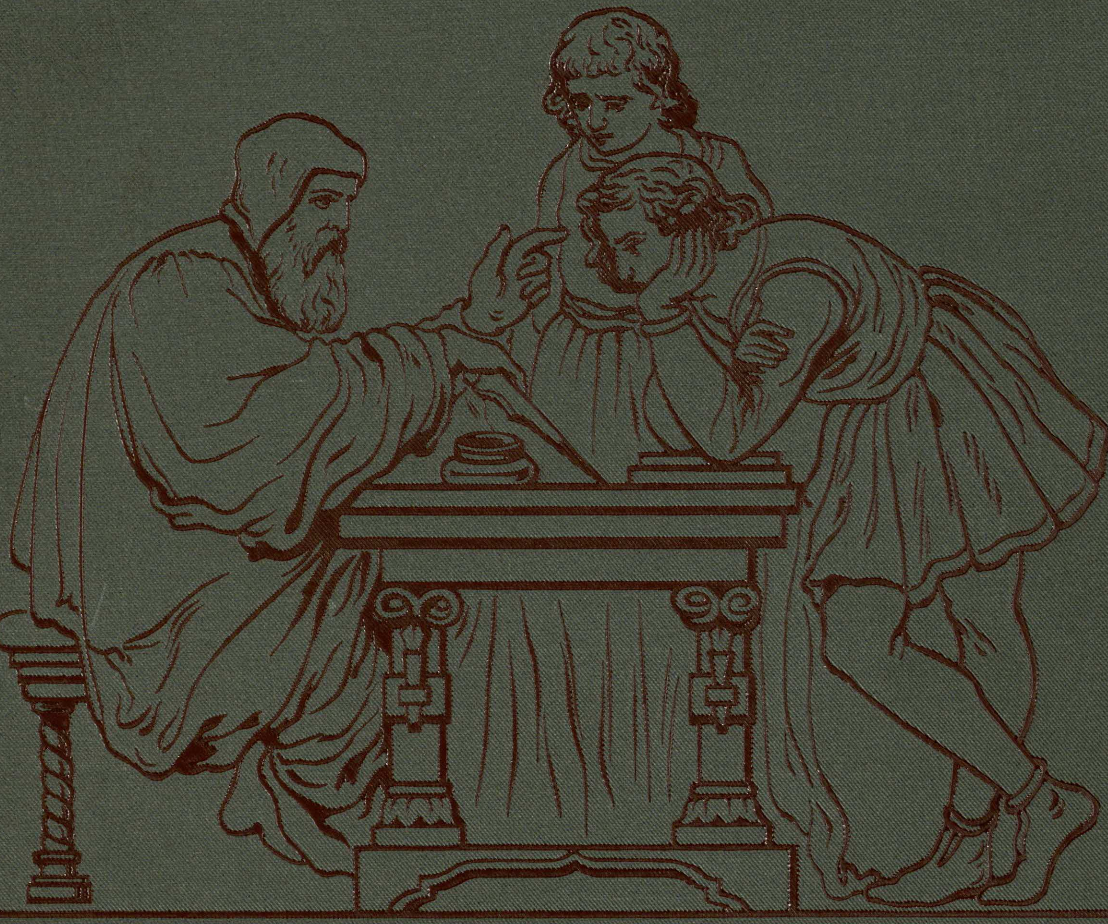
·POYNTER'S·SOUTH·KENSINGTON·DRAWING·BOOK·

ELEMENTARY ·PERSPECTIVE·DRAWING·

·BY·S·J·CARTLIDGE·F·R·HIST·S·



SANCTIONED·BY·THE·COMMITTEE·OF·COUNCIL·ON·EDUCATION

POYNTER'S SOUTH KENSINGTON DRAWING BOOK.

UNDER THE SANCTION OF
THE LORDS OF THE COMMITTEE OF COUNCIL ON EDUCATION.

ELEMENTARY PERSPECTIVE DRAWING,

INCLUDING THE

PROJECTION OF SHADOWS

AND

REFLECTIONS.

BY

S. J. CARTLIDGE, F.R.HIST.S.

HEAD MASTER, GOVERNMENT SCHOOL OF ART, HANLEY;
FORMERLY LECTURER AT THE NATIONAL ART TRAINING SCHOOLS, SOUTH KENSINGTON.

"AS ART SHALL ADVANCE, ITS POWERS WILL BE STILL MORE AND MORE FIXED BY RULES."—Sir Joshua Reynolds.

LONDON: BLACKIE & SON, PUBLISHERS;
GLASGOW, EDINBURGH, AND DUBLIN.
1884.

LUCEM·LIBRIS DISSEMINAMUS

BLACKIE AND SON,
GLASGOW, LONDON, AND DUBLIN.

ELEMENTARY PERSPECTIVE DRAWING.

PREFACE.

This Work is intended as a first book of Perspective, and its aim is to place the subject before the student in as simple a manner as possible.

The usual plan of dividing the science into "*Parallel*" and "*Angular*" Perspective has been abandoned, as unnecessarily protracting the labour of the student, who is, in the following lessons, gradually led from the finding of an isolated point, through lines, figures and solids in easy positions, to the more complex and difficult branches of the science. From the adoption of this plan it will be found that much of the matter commonly contained in elementary books on the subject has been advantageously omitted.

The Work is in Four Parts; the first two of which are specially adapted to those intending to enter the Second Grade Examination of the Science and Art Department. The Third Part treats of Accidental Vanishing Points, and is an easy introduction to higher Perspective, also to the study of Shadows and Reflections comprised in the Fourth Part.

S. J. CARTLIDGE.

Both the general plan and details of this Work have been seen and approved by me as it has progressed. The system adopted is similar to that used in the Class Lectures given by Mr. Cartlidge in the National Art Training School, with results that were entirely satisfactory.

JOHN C. L. SPARKES,
Principal of the National Art Training School,
South Kensington Museum.

GENERAL INSTRUCTIONS FOR PERSPECTIVE DRAWING.

The Plates should be carefully studied by reference to the text, and each problem drawn out in lead pencil, in order to ensure the complete comprehension of every point in the diagrams. The exercise appended to the problem should then be worked, and it need hardly be said that each problem and exercise should be thoroughly understood before proceeding with the next.

The method of study recommended is to make the copy *larger* than the diagram, but this is not absolutely necessary.

The instruments required are a T-square, pair of 5-inch compasses, and two set-squares, one having an angle of 45 degrees and the other an angle of 60 degrees, also a protractor scale for the measurement of angles. With regard to pencils, HH should be used for "*working*" lines, and HB for the "*figure*" lines.

Smooth paper is the most suitable, and care must be taken not to pierce through its surface with either pencils or instruments. It should be pinned down upon a drawing-board to secure an even surface, without which an accurate drawing is scarcely possible.

The progress of the student will be much facilitated by consecutively lettering the lines and points in his drawing, and by adhering to the use of the varieties of line employed in the plates, to distinguish the different parts of each problem.

ABBREVIATIONS USED IN THE WORK.

P.P. — Picture Plane.
G.P. — Ground Plane.
V.P. — Vanishing Point.
V.L. — Vanishing Line.
H.L. — Horizontal Line.
G.L. — Ground Line.
M.P. — Measuring Point.
C.V. — Centre of Vision.
P.D. — Point of Distance.
R.A. — Right Angles.
I.L. — Intersecting Line.
I.P. — Intersecting Point.
V. — Vertical.
A. — Accidental.
O. — Original.
I. — Inclination.
Van. — Vanishing.
Par. — Parallel.

ELEMENTARY PERSPECTIVE DRAWING.

INTRODUCTION.

PERSPECTIVE is a science which teaches us to *see* correctly, and enables us to represent the *appearance* of anything we may wish to draw. If we carefully observe young children, we find that the power of seeing correctly is the result of education. A child is a long time before it has a just idea of distance. This may be observed in many ways; for example, a child on trying to walk to an object, finds that a certain number of steps must be made, in order to reach it; and, if we change the *position* of the object, by placing it further from the child, it discovers that more steps are required: and often appears considerably puzzled at the difference.

We also find that the slightest alteration of *position* will change the *appearance* of an object, and the eye of the child, after some perplexing trials, becomes slowly accustomed to the changes of appearance. Thus learning by experience, the child applies its knowledge to everything it sees, and so gradually acquires (just as it gradually acquires the power of speech) that power of sight which is common to us all. If we wish to go beyond this common power of sight we must learn Perspective, which will enable us to detect the slightest variation in the appearance of anything we see.

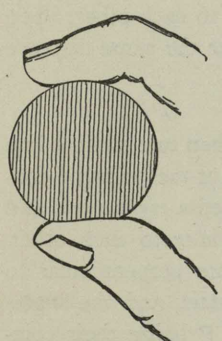


Fig. 1.

That the appearance of an object varies according to the position in which it is placed, can be readily proved. For illustration we will take a penny, or any other coin, the actual shape of which is a perfect round; or, strictly speaking, a circle. If we take the coin between the thumb and the first finger, holding it in an upright position, and exactly facing the eye (as in **Fig. 1**) it appears of its true form, viz. a circle. If we alter its position, balancing it upon the thumb, in a level position, with its edge directly opposite the eye (as in **Fig. 2**), its appearance is changed, and what we know to be really a circle, *appears* to us as a straight line. Now, still balancing the coin upon the thumb, but *changing its position with regard to the eye*, by holding it a little lower than in the last position, that is slightly beneath the level of the eye (as in **Fig. 3**); we see both the edge and the surface, the coin now appearing neither a circle nor a straight line but a curved figure of an elliptical form. Thus the same coin held in three different positions has assumed three different *shapes*.

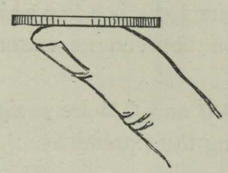


Fig. 2.

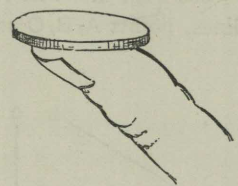


Fig. 3.

Let us take *two* coins of the same size, holding (in the position shown at **Fig. 1**) one in each hand. Now, closing one eye (which will make the experiment more clear), hold one coin out at arm's length, and the other at the distance of about a foot from the eye. On comparing them, we find that the coin which is further from the eye appears *less* than the nearer one. We know that the coins are really equal in size, yet one *appears* smaller than the other.

We thus see that when we change the *position* of an object, we have as a consequence a change of *appearance;* also that the change of appearance may affect both the *shape* and the *size* of the object.

These diversities of appearance may be remarked in everything around us. We can observe them in the street, by looking at a building from different points of view, or by comparing the apparent sizes of the street lamps; in the railway station, by watching the arriving or departing train; and at sea, by noticing the vessels as they approach, or as they retire, ultimately vanishing from our sight in that line where the sea and sky appear to meet.

All variations of appearance are in strict accordance with the laws of **GEOMETRY** and **OPTICS.**

GEOMETRY is the science which governs the actual sizes and shapes of objects; and it is quite obvious that we must know something of it to be able to understand the exact form of any object we may wish to draw. Indeed it will be quite impossible to proceed with the study of Perspective without this geometrical knowledge. The Geometry essential to a thorough comprehension of the diagrams in this book, we will endeavour to master as we proceed; but it will be advisable to study from time to time, some work on Elementary Geometry, both Plane, and Solid; the latter being especially useful.

Let us then commit the following Geometrical definitions to memory:—

Definition 1. A POINT is that which denotes *position.* It may be either a dot, the end or extremity of a line, or the crossing or meeting of lines.

Definition 2. A LINE is that which has *direction* or *length.* The different styles of line which will be used in the following diagrams are as shown at **Fig. III. Plate I.** a fine line **Z**, a strong line **Y**, a dotted line **X**, a chain dotted line **W**, and a double chain dotted line **V**.

Definition 3. A RIGHT or STRAIGHT LINE is a line which continues in the same direction between its extreme points or ends.

See **Fig. 4.** The two lines **A B** and **C D** are straight lines, points **A, B, C** and **D**, being the extremities.

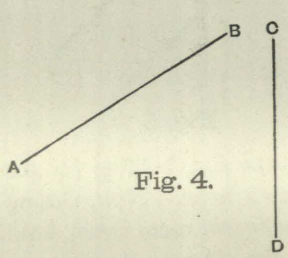


Fig. 4.

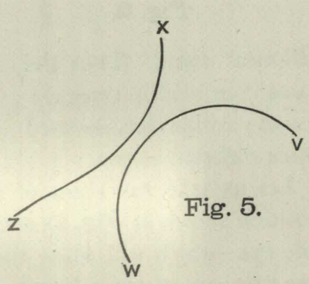


Fig. 5.

Definition 4. A CURVED LINE is a line which continually changes its direction.

See **Fig. 5.** The two lines **Z X** and **V W** are curved lines.

Definition 5. An ANGLE. When two straight lines incline towards each other and meet in a point, they form an *angle.* See **Fig. 6.**

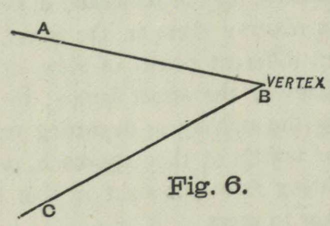


Fig. 6.

The point in which the lines meet is called the VERTEX of the angle. The size of the angle depends upon the *degree of opening* between the lines which form it.

By opening a pair of compasses we may gain a good idea of the nature of an angle. The angle formed by the legs of the compasses becomes greater as the opening widens. An angle is usually marked by three letters, as **A B C**, the *middle letter denoting the* VERTEX of the angle.

Definition 6. A PLANE is a *surface* which is perfectly *even* and *flat.*

To use a familiar illustration, a Plane is like the surface of a sheet of plate glass. Recollect particularly, that *a surface which is at all curved, is not a plane.*

Definition 7. HORIZONTAL means perfectly *level*, like the surface of still water.

We must be careful to understand exactly the difference between the terms "level", and "even" or "flat." A surface may be "*even*" or "*flat*", without being "*level.*" Thus the wall is even and flat, but it is *upright*, not *level.* LEVEL means a fixed, constant position.

Definition 8. VERTICAL means perfectly *upright.*

If we attach a piece of thread to a weight, a small piece of lead for example, and hold the thread with the lead hanging downwards, the thread will fall in an upright, or *vertical* position. The surface of a flat sheet of glass in an upright position would be a *Vertical Plane.*

Definition 9. PARALLEL. Lines are said to be parallel to each other when they are throughout their whole lengths the *same distance apart.* See **Fig. 7.**

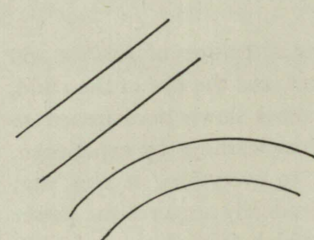


Fig. 7.

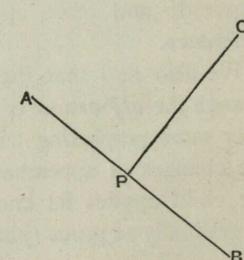


Fig. 8.

In the same way *Planes* are parallel to each other, when they are throughout their *entire surfaces* the same distance apart.

Definition 10. PERPENDICULAR. When one straight line meeting another, makes the *angles* at the point of contact *equal*, each of the angles is called a *right angle*, and the lines are said to be *perpendicular* to each other. See **Fig. 8.** Lines **A B** and **P C** are perpendicular to each other. **P** is the point of contact, and the angles **C P A** and **C P B** are right angles, **P** being their common vertex. See **Definition 5.**

Remember especially that *Perpendicular* and *Vertical* have not the same meaning. *Vertical* means an *unvarying upright position. Perpendicular* means that one line or plane meets another line or plane at *right angles.*

Definition 11. A CIRCLE is a plane figure contained by one line, which is called the *circumference.* The circumference is in every part equidistant from the *centre* of the circle. An *arc* of a circle is any portion of the circumference. A straight line passing through the centre of the circle and meeting the circumference on each side is called a *diameter.* A straight line drawn from the centre of the circle to any part of the circumference is called a *radius.* See **Fig. 9.**

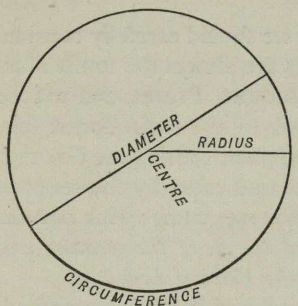


Fig. 9.

The circumference of a circle is supposed to contain 360 *equal parts* called *degrees.* A small cypher is made use of to signify degrees; thus sixty degrees is written 60°.

If we wish to draw a line making a certain angle with another line, the method is as follows. Let it be required to draw a line from X (**Fig. 10**) to be inclined at an angle of 70° with line A B. Placing the point of the compasses in

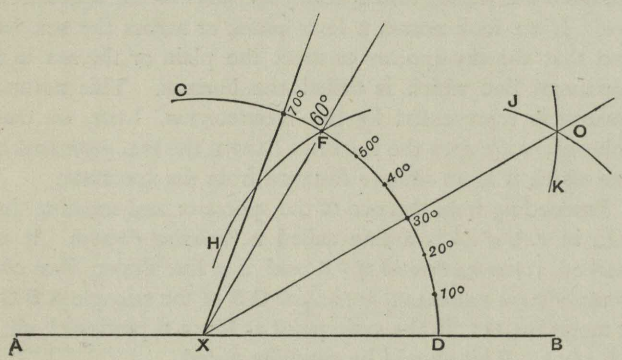


Fig. 10.

X, with any radius or opening, less than X B, describe the arc C which meets line A B in D. Then with the point of the compasses in D (*keeping the same radius as for arc* C) describe arc H, which meets arc C in F. A straight line drawn from X through F will be at an angle of *60°* to A B. If we divide arc D F (which contains 60°) into two equal parts we shall have two portions of 30°. From F and D as centre, with any radius describe arcs J and K which meet in O.

Join O to X, line O X is at an angle of *30°* to A B. If we divide arc D 30° into three equal parts—*by trial*—we shall have three divisions of 10° each. This may be repeated from 30° to F and we shall have arc D F divided into six equal parts of 10°. If we measure one of these divisions of 10°, and set it off *from* F on the side of the arc towards C, we have 10° + 60° = 70°. Draw a straight line from X through 70°—it is the line required. This method will do for any angle, remembering to start with 60°, which is *always found* by *setting off the radius* of the circle *upon the circumference.*

Definition 12. A TRIANGLE is a figure inclosed by *three straight lines* which are called the *sides* of the figure.

Triangles may be:—

a. ISOSCELES, having *two equal sides* and *two equal angles.*

b. EQUILATERAL, having *three equal sides* and *three equal angles.*

c. SCALENE, having *unequal sides* and *unequal angles.*

Definition 13. A RECTANGLE is a *four-sided* figure, having its *opposite* sides *parallel* to each other, and its angles *right angles.*

Definition 14. The ALTITUDE of a figure is the straight line drawn from its *highest point* at *right angles* to its base.

OPTICS, the science of sight, gives us the following laws:—

1. That we see by the agency of *light.*
2. That light passes from objects to our eyes.
3. That light travels in *straight lines* which are called VISUAL RAYS.

PLATE I.

THE word PERSPECTIVE means to *see through;* and the science is so named because anything drawn in Perspective would coincide with the appearance of the same thing seen through and traced on an *upright*, *transparent*, *flat surface*, which is called the PICTURE PLANE. In **Plate I. Fig. I.** the PICTURE PLANE is represented by the rectangle W X Y Z. Although the PICTURE PLANE is here shown as a rectangle, it may be of *any shape* and of *any size.*

The OBSERVER is at S, looking through the PICTURE PLANE at the cross R C O H. The observer is standing upon a *horizontal* surface which is called the GROUND PLANE. If we are in a room, the window may be called a PICTURE PLANE, and the floor a GROUND PLANE.

The PICTURE PLANE rests, as it were, upon the GROUND PLANE in a line which passes from Y to Z. The two planes meet or *intersect* in this line, which is called the GROUND LINE. The GROUND LINE is sometimes called the PICTURE LINE, or the MEASURING LINE.

The VISUAL RAYS, by means of which the observer sees the cross, will, in their course from it to her eye, *pass through* the PICTURE PLANE. These VISUAL RAYS will intersect the PICTURE PLANE in a number of points, and if we mark the true positions of these points, the result will be a Perspective image of the cross.

The rays are shown passing from the cross to the eye of the observer and meeting the PICTURE PLANE in points r c o h, r being joined to o, and c to h; we have the Perspective image of the cross as it would appear to the observer at S. Of course an infinite number of rays proceed from the cross to the eye of the observer; but it is quite evident that we need only consider those proceeding from the *extremities* of the object.

But how are we to fix the exact positions of these points in which the VISUAL RAYS intersect the PICTURE PLANE.

In **Fig. II. Plate I.** we have the PICTURE PLANE and the GROUND PLANE with a triangle A B C lying upon the GROUND PLANE. The perspective image or representation is shown upon the PICTURE PLANE at a b c.

Let us see how the intersection of the VISUAL RAY, passing from the corner of the triangle marked B, is determined.

A line is drawn upon the GROUND PLANE from B to the feet of the observer or spectator, or more strictly, to a point upon the GROUND PLANE *vertically below the spectator's eye.* This chain dotted line is, *every part of it, vertically beneath Visual Ray B;* and it intersects the GROUND LINE in point 3. Point 3 is in the chain dotted line, therefore it is *vertically beneath Visual Ray B.* Point 3 is also in the PICTURE PLANE, because the GROUND LINE is the intersection of the GROUND PLANE and the PICTURE PLANE; therefore, if we draw a vertical line from 3 *to meet Visual Ray B,* we shall find the actual intersection of *Visual Ray B* with the PICTURE PLANE in point b.

The chain dotted line drawn from B to the spectator's feet is called a GROUND INTERSECTION, inasmuch as it is the intersection with the GROUND PLANE of a plane passing through B and the spectator's eye. As the GROUND INTERSECTIONS are of great importance we should carefully consider and realize what they are. They are always the result of an *intersection* of a *plane* with the GROUND PLANE, and will be distinguished throughout this book by the chain dotted line. In the particular case before us the plane causing the GROUND INTERSECTION is a *Vertical Plane* (indicated by the series of Vertical Lines passing from B to the spectator) which *contains Visual Ray B, passes through* B *and the eye of the spectator* and *intersects the* PICTURE PLANE *in the Vertical Line* 3 b.

The intersections of A and C are found by the same method, viz., by *Visual Ray A* and its GROUND INTERSECTION, and *Visual Ray C* and its GROUND INTERSECTION.

There will also be noticed a point upon the GROUND LINE marked *Point of Contact,* which is the point where side B C extended or produced, comes into contact with the PICTURE PLANE.

The point upon the PICTURE PLANE marked C.V. is the CENTRE OF VISION, which is *exactly opposite* the spectator's eye. The ray joining C.V. to the eye of the spectator is the *Principal* or *Central Visual Ray.*

Passing through C.V. is a line marked H.L., signifying HORIZONTAL LINE, which marks the level of the spectator's eye. If we look across a level plain, or across the sea, we find that the sky appears to meet the plain or the sea in a horizontal line which is called the horizon. This natural horizon is represented by the HORIZONTAL LINE, so that although *really upon* the PICTURE PLANE, the H.L. *represents* a line which is at an infinite distance from the spectator.

Proceeding from the eye of the spectator and meeting the H.L. in *V.P. of c b* is a line called a *Vanishing Parallel.* It is marked *Vanishing Parallel of C B,* and is a line drawn *from the spectator's eye* PARALLEL *to the side* C B of the triangle A B C. It meets the H.L. in the same point as line c b produced, viz., *V.P. of c b.* This should be carefully noted.

To repeat:—

Vanishing Parallel of C B is *actually parallel* to the original side C B of the triangle.

It is produced to meet the PICTURE PLANE in the H.L..

It meets the H.L. in *V.P. of c b.*

Line c b is the perspective representation of C B.

Line c b produced meets the H.L. in *V.P. of c b,* so that *Vanishing Parallel of C B,* and c b produced, meet the H.L. in the *same point.*

When we compared our two coins, that which was further from the eye appeared the smaller. Let us glance at an illustration of the same fact worked out by means of the PICTURE PLANE and the VISUAL RAYS. In **Fig. IV. Plate I.,** line P.P. represents the PICTURE PLANE seen from the side. E is the position of the spectator's eye. H I and J K are two lines of *equal length* but at *different distances* from the spectator's eye. Rays drawn from H and I to the spectator's eye at E cut the PICTURE PLANE in h and i. h i is the perspective representation of the line H I. Rays drawn from J and K to E intersect the PICTURE PLANE in j and k. j k is the perspective representation of J K, and, J K being further from the spectator, appears less than H I which is nearer to the spectator.

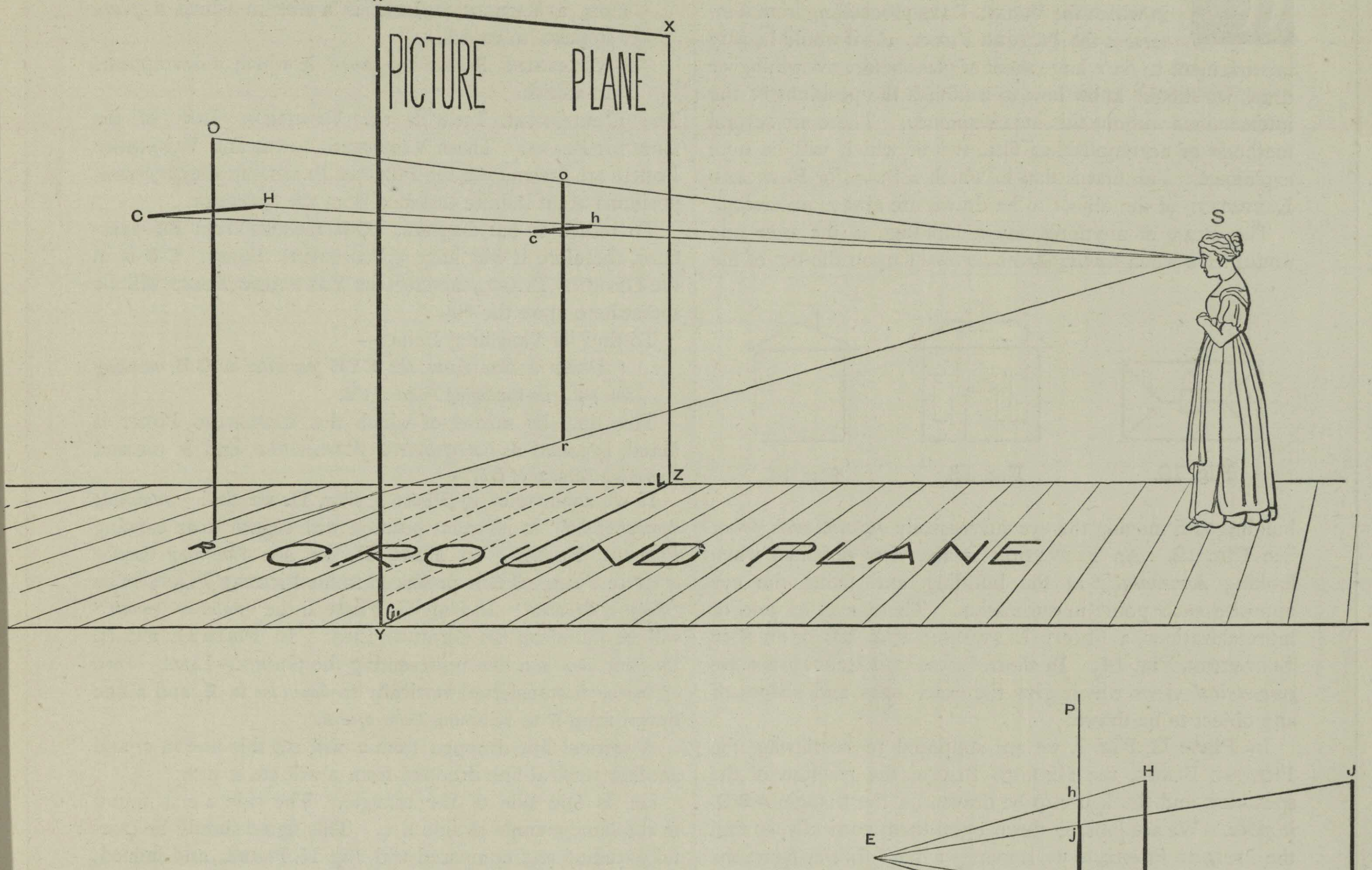


Fig. I.

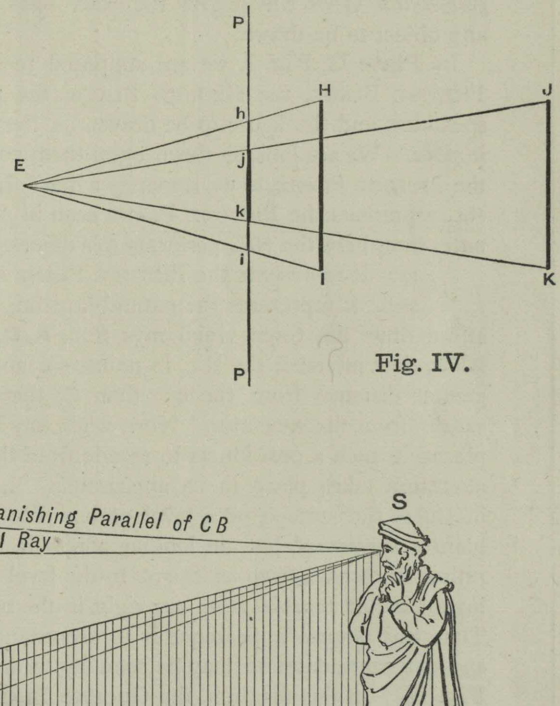


Fig. IV.

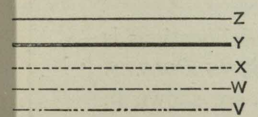


Fig. III.

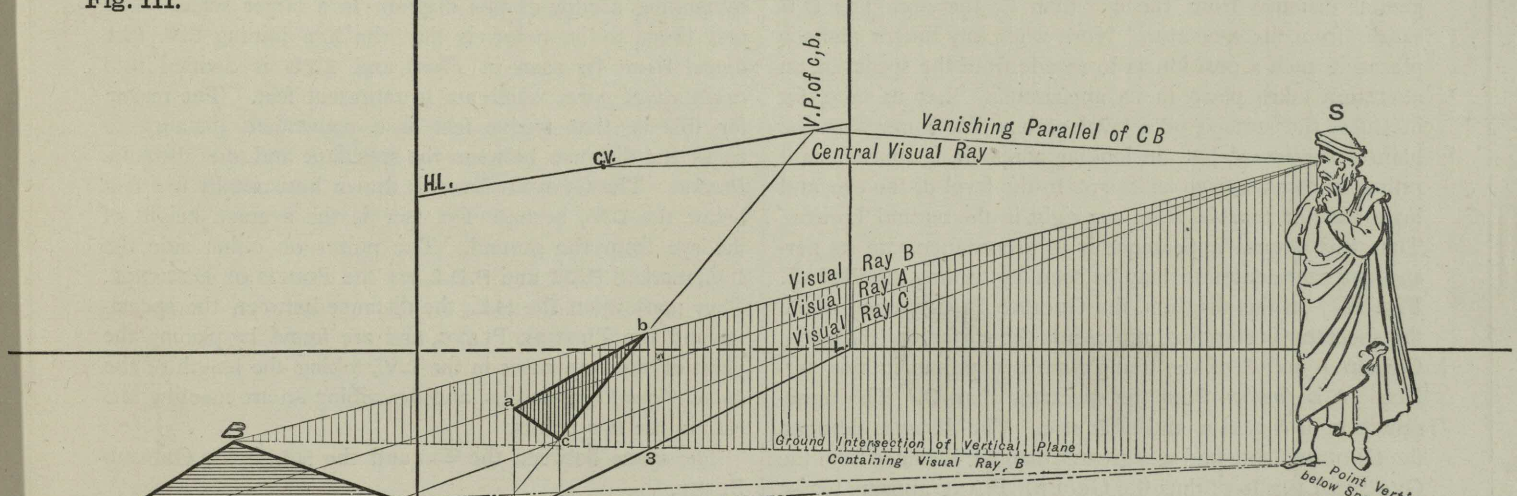


Fig. II.

Plate II.

We have seen that the perspective representation of an object is found by establishing the points in which the Visual Rays proceeding from it intersect the Picture Plane. As it would be very inconvenient to fix a large sheet of glass before everything we draw, we should know how to establish the positions of the intersections without this inconvenience. There are several methods of accomplishing this, two of which will be here explained. The first is that in which a Plan, or Plan and Elevation of the object to be drawn are always necessary.

The Plan of anything, say a building, is the view one would have when *looking down vertically* upon the top of the

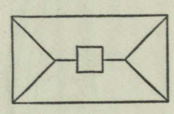


Fig. 12.

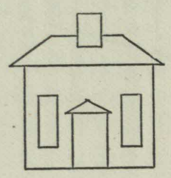


Fig. 13.

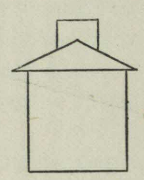


Fig. 14.

building and moving the eye successively *opposite each point.* See **Fig. 12.** An Elevation is the view obtained when looking *horizontally* at the building, and fixing the eye opposite each point in succession. There may be one or more elevations, a Front Elevation, **Fig. 13**; or an End Elevation, **Fig. 14.** In short, Plans and Elevations are *geometrical* views which give the exact sizes and shapes of any object to be drawn.

In **Plate II. Fig. I.** we are supposed to be viewing the Picture Plane, the Ground Plane, the position of the spectator, and the figure to be drawn, *i.e.* the triangle A B C, in *plan.* We are looking down upon them vertically, so that the Picture Plane, to us, is merely a line. In this figure the H.L. represents the Picture Plane seen in *plan.* We must note, that, here the H.L. performs *two* offices.

1st. It represents the Picture Plane seen in plan.

2nd. It represents the natural horizon.

If we draw the three visual rays from A, B, and C to the EYE, they intersect the H.L. in points 1, 2, and 3. B is at a greater distance from the eye than C, therefore line C B *recedes* from the spectator. Now, when any line or plane is placed in such a position as to recede from the spectator, an alteration takes place in its appearance. Let us take, for instance, the surface of a level plain. We know that the plain is *horizontal*, but, on looking across it, it appears, as it retires or recedes from us, to *rise* to the level of the eye, and loses itself or *vanishes* from our sight in the natural horizon. That this natural appearance is in accordance with its perspective representation may be seen by turning to **Plate I. Fig. II.** In this diagram, the Ground Plane, upon which the spectator is standing, represents the surface of the plain. Corners A and C of the triangle are two points in this surface. A is *further* from the spectator than C. The representation of A is at a, and of C at c. The distance between the Ground Line and a is greater than the distance from the Ground Line to c: thus the Ground Plane appears to rise as it recedes.

The term "Vanishing," so much used in Perspective, arises from the fact that a line or plane which recedes from us will, if extended far enough, ultimately *vanish* from our sight. A *Plane* will vanish in a *line* and a *line* will vanish in a *point.*

Thus, a Vanishing Line is a *line* in which a *plane* appears to vanish.

A Vanishing Point is a *point* in which a *line* appears to vanish.

The Horizontal Line is the Vanishing Line of the Ground Plane. These Vanishing Lines and Vanishing Points are drawn *upon* the Picture Plane but they *represent* positions at an infinite distance from the spectator.

To return to our diagram. C B recedes from the spectator, therefore it will have a Vanishing Point. C B is in the Ground Plane, therefore its Vanishing Point will be somewhere upon the H.L..

To find its Vanishing Point:—

Draw a line from the EYE *parallel to* C B, *meeting the* H.L. *in Vanishing Point of C B.*

This line, by means of which the Vanishing Point is found, is called a *VANISHING PARALLEL*, and is marked *Vanishing Parallel of C B.*

If we again refer to **Plate I. Fig. II.** we find *"Vanishing Parallel of C.B."* in its exact position with regard to an existing Picture Plane. In **Plate II.** we see *Vanishing Parallel of C B* in *Plan.* B C is produced to the Picture Plane in its *"Point of Contact."* In **Fig. II. Plate I.** the *"Point of Contact"* will be found on the Ground Line. In **Plate II. Fig. I.** the *Base Line* is a line representing the Ground Line. *Point of Contact* is transferred vertically to *Base Line* in P, and a line drawn from P to *Vanishing Point of C B.*

A vertical line dropped from 2 will cut this line in c, and another vertical line dropped from 3 will cut it in b.

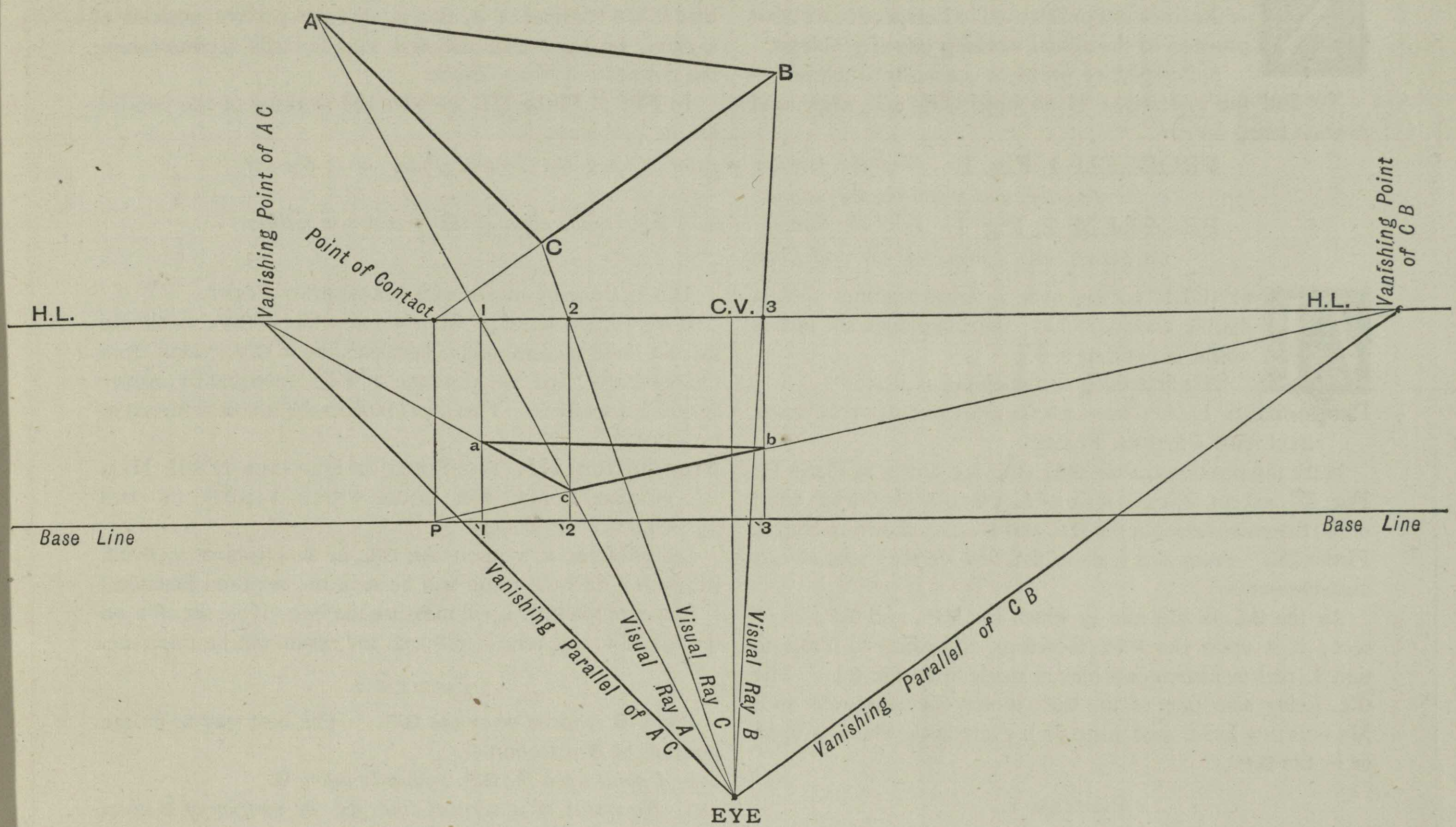
c.b. is one side of the triangle. The side a c is found in the same manner as side b, c. This figure should be carefully studied and compared with Fig. II. Plate I., and indeed, it would be well to copy the diagram as a means of more thoroughly impressing it upon the memory.

The second method of making Perspective delineations is that of using Vanishing Points and *MEASURING POINTS.* In **Fig. II. Plate II.** the C.V., H.L., and G.L. are shown in their relative positions. We must now commence drawing by making a copy of this diagram to a larger scale. The first thing to be noted is that the line joining C.V. (the *Central Visual Ray* seen in *Plan*) and EYE is divided into *twelve equal parts*, which are to represent feet. The reason for this is, that twelve feet is a convenient distance to fix as the distance between the spectator and the Picture Plane. The Ground Line is drawn horizontally five feet below the C.V., because five feet is the average height of the eye from the ground. The points on either side the C.V., marked P.D.1 and P.D.2, are the Points of Distance. They mark upon the H.L., the distance between the spectator and the Picture Plane, and are found by placing the point of the compasses in the C.V., taking the length of the *Central Visual Ray* as radius, and describing an arc meeting the H.L. in the two points.

The space between the G.L. and the H.L. is the Ground Plane.

A diagram similar to this will be required for each Perspective drawing, for which *any scale* may be taken, but the *Central Visual Ray* must be 12′ long, and the H.L. and G.L. 5′ apart.

Fig. I.


A
B
C
Vanishing Point of A C
Point of Contact
1
2
C.V.
3
H.L.
H.L.
Vanishing Point of C B
a
b
c
Base Line
P
`1
`2
`3
Base Line
Vanishing Parallel of A C
Visual Ray A
Visual Ray C
Visual Ray B
Vanishing Parallel of C B
EYE

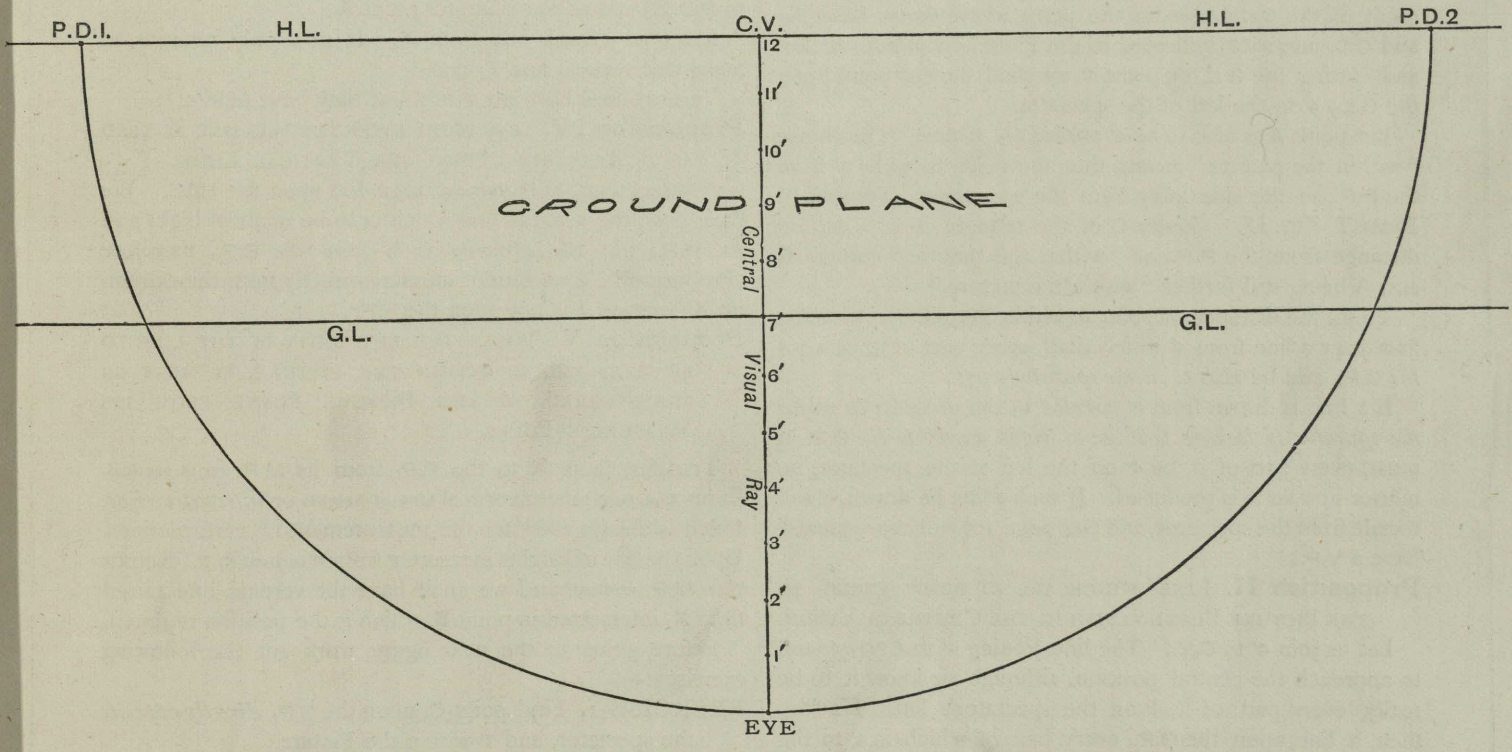

P.D.I.
H.L.
C.V.
H.L.
P.D.2
12
11′
10′
GROUND 9′ PLANE
Central
8′
G.L.
7′
G.L.
Visual
6′
5′
Ray
4′
3′
2′
1′
EYE

Fig. II.

PLATE III.

NOW that we understand something of the positions of the lines and points used in Perspective, we must proceed to the actual working out of problems.

A Perspective problem is usually as follows:—

To find the *appearance* of an object, its *size*, *shape*, and *position* being given.

Objects are bounded by *lines*, lines terminate in *points:* so that if we commence by finding the perspective position of a *point*, we shall take the first step towards accomplishing the delineation of an object.

In **Fig. I. Plate III.** we have the working of the two following problems.

PROBLEM 1, Fig. I. *Find the position of point* A *upon the Ground Plane,* 4′ *to the left of the spectator, and* 3′ *within the picture.*

PROBLEM 2, Fig. I. *Find the position of point* B, 5′ *to the right of the spectator,* 4′ *within the picture, and* 7′ *above the Ground Plane.*

A SINGLE accent over a figure signifies feet, a double accent inches; thus, two feet six inches would be written 2′·6″.

The first thing to remember is that:—

Proposition I. ALL REAL MEASUREMENTS MUST BE MADE UPON THE PICTURE PLANE.

Mark the positions of the G.L., H.L., &c. &c. as in **Plate II. Fig. II.**, except that the scale of feet is to be at the left hand of the diagram, between the G.L. and H.L., as shown in **Fig. I. Plate III.** From this scale of five feet we may take all our measurements.

As the G.L. is the line in which the P.P., and G.P., intersect; it is upon the P.P., therefore, according to Proposition I., real measurements may be made upon the G.L.. The G.L. being also part of the G.P., it may be employed as a MEASURING LINE (see page 8) for anything which is upon or in the G.P..

PROBLEM 1.

Point A is to be 4′ to the left of the spectator. *The Central Visual Ray indicates the exact central position of the spectator*, and if we measure, upon the G.L., 4′ on the left of the *Central Visual Ray*, by taking a radius or opening of 4′, and placing the point of the compasses in the point where *Central Visual Ray* and G.L. intersect (indicated in the Plate by a minute circle), and cutting the G.L. in point 4′, we shall have a point upon the G.L., 4′ to the left of the spectator.

But point A is also to be 3′ *within the picture.* The phrase "within the picture" means that the point is to be 3′ from the P.P. on the side away from the spectator. Turning to **Plate I. Fig. II.** Corner C of the triangle is at a certain distance from the P.P., or "within the picture," corners B and A being still further "within the picture."

To fix the distance of point A within the picture, we must first draw a line from 4′ which shall, every part of it, be *upon the* G.P., and be also 4′ *on the spectator's left.*

If a line is drawn from 4′ *parallel* to the *direction in which the spectator is looking*, that is, *at right angles to the* P.P., it must, every part of it, be 4′ on the left of the spectator, no matter how far it is produced. If such a line be drawn, it will recede from the spectator, and (see page 10) will consequently have a V.P..

Proposition II. LINES WHICH ARE AT RIGHT ANGLES TO THE PICTURE PLANE VANISH IN THE CENTRE OF VISION.

Let us join 4′ to C.V.. The line joining 4′ to C.V. *appears* to approach the central position, although we know it to be really, every part of it, 4′ on the spectator's left. Having, then, a line upon the G.P., every part of which is 4′ to the left, we have only to *measure* upon this line, a distance of 3′ within the picture.

This is done by means of a MEASURING POINT.

If we were to set off, 3′ from 4′, upon line 4′ C.V., we should make a false measurement, because line 4′ C.V. *recedes* from the spectator, and the distance 3′ will consequently *appear less* than it really is. This fact involves the use of a MEASURING POINT.

Proposition III. THE POINT OF DISTANCE IS THE MEASURING POINT FOR LINES WHICH VANISH IN THE CENTRE OF VISION.

Set off from 4′, 3′ upon the G.L. in 3′. Join 3′ to P.D.1. (the M.P.), and the result will be A in the required position.

Either of the P.D.'s. will measure the line. If we set off 3′ on the *left* of 4′, and join it to P.D.2. the result will be the same.

PROBLEM 2.

Point B is to be *above* the G.P.. The best way to fix the position of B is to find—

1st. *A point upon the* G.P. *vertically below* B.

2nd. *By means of a vertical line, fix the position of* B *above the* G.P..

Find, then, the position of X upon the G.P. vertically below B, by setting off 5′ on the right of the *Central Visual Ray*—joining 5′ to C.V.—measuring 4′ from 5′ in 4′—and joining 4′ to P.D.2.: just as was done for point A.

Erect a vertical line from X. It now only remains to make this vertical line 7′ high.

Vertical lines have no V.P.'s. but they have M.P.'s.

Proposition IV. *ANY* POINT UPON THE H.L. MAY BE USED AS A MEASURING POINT FOR VERTICAL LINES.

Assume, then, M.P. Vertical, *anywhere* upon the H.L.. We have now, the vertical line which is to be made 7′ high, and its M.P., but its *extremity* is 4′ *from the* P.P., therefore (Proposition I.), we cannot measure directly from the extremity X because it is not upon the P.P..

Proposition V. WHEN THE EXTREMITY OF THE LINE TO BE MEASURED IS WITHIN THE PICTURE, IT MUST BE TRANSFERRED TO THE PICTURE PLANE FROM ITS MEASURING POINT.

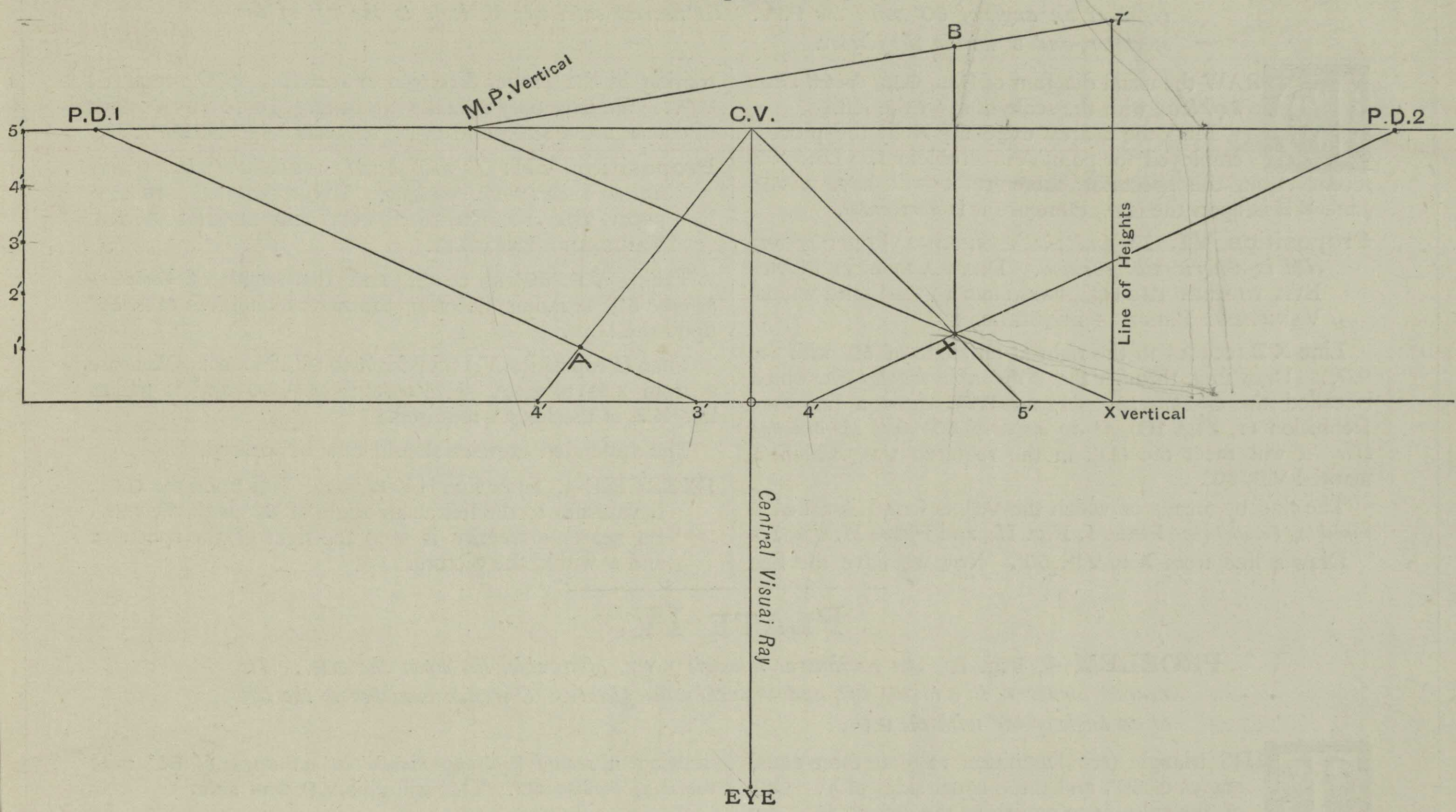
Transfer, then, X to the P.P. from its M.P. in X vertical. From X vertical raise a vertical Line of heights or *intersecting line*, which is always used for the measurement of vertical lines. Upon the line of heights measure 7′ from X vertical in 7′. Return 7′ to M.P. vertical, and we shall have the vertical line raised from X, intersected in point B, which is the position required.

Before going to the next figure, work out the following exercises:—

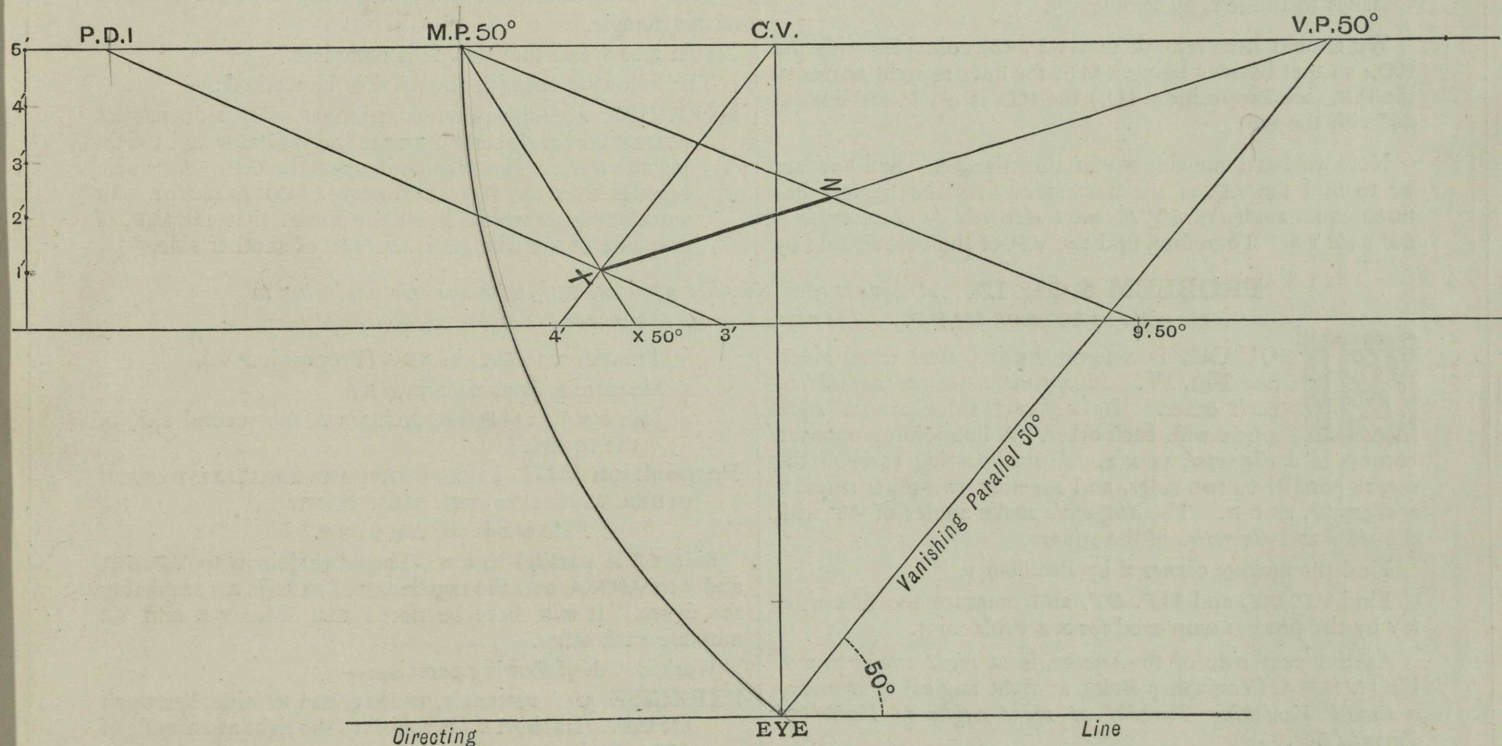
EXERCISE 1. Find point C, upon the G.P., *directly opposite* the spectator, and 7′ within the Picture.

EXERCISE 2. Find point D, 12′ above the G.P., 9′ within the picture, and 1′ to the *left* of the spectator.

Fig. I.



B
7′
M.P. Vertical
P.D.1
C.V.
P.D.2
5′
4′
3′
2′
1′
Line of Heights
A
4′
3′
4′
5′
X vertical
Central Visual Ray
EYE



P.D.1
M.P. 50°
C.V.
V.P. 50°
5′
4′
3′
2′
1′
N
4′
X 50°
3′
9′.50°
Vanishing Parallel 50°
50°
Directing
EYE
Line

Fig. II.

PLATE III.—*Continued.*

PROBLEM 3, Fig. II. *A straight line* X Z, 9′ *long, lies upon the* G.P.. *It recedes to the right, at an angle of* 50° *with the* P.P.. *Its nearest extremity* X *is* 4′ *to the left of the spectator, and* 3′ *within the picture.*

DRAW the usual diagram of H.L., G.L., *Central Visual Ray*, &c., with the scale of 5′ on the left.

Find the nearest extremity X by the process employed for point A in Problem I. Line X Z recedes from the spectator, therefore it will have a V.P. Line X Z is upon the G.P., therefore it is *horizontal.*

Proposition VI. *To find the* VANISHING POINT *of any line in a horizontal position.* DRAW A LINE FROM THE EYE, TO MEET THE H.L., PARALLEL TO THE LINE WHOSE VANISHING POINT IS REQUIRED.

Line X Z recedes to the right at an angle of 50° with the P.P.. Draw first, through the EYE, a *horizontal line* which is called the *Directing Line.* From EYE draw a line (see Definition 11, Fig. 10), *at an angle of* 50° *with the Directing Line:* it will meet the H.L. in the required V.P., which is marked V.P. 50°.

The line by means of which the V.P. is found is called a *Vanishing Parallel* (see Plate I. Fig. II., and Plate II. Fig I.).

Draw a line from X to V.P. 50°. Now we have the line retiring in the proper direction or receding to the required V.P.. We have only to make it 9′ long. To do this we must of course use a M.P.; and every V.P. has a special M.P..

Proposition VII. *To find the* MEASURING POINT *of any line in a horizontal position.* UPON THE H.L., SET OFF, FROM THE VANISHING POINT, THE LENGTH OF THE VANISHING PARALLEL.

Taking V.P. 50° as centre, and the length of *Vanishing Parallel* 50° as radius, describe the arc which gives M.P. 50° upon the H.L..

Then (Proposition V.) transfer X to G.L. in X 50°. Measure 9′, from X 50° to 9′.50°. Join 9′.50° to M.P. 50° and it makes line X Z of the length required.

The appended exercise should now be worked.

EXERCISE 3. Draw line H K 10′ long. It is upon the G.P.. It vanishes to the left at an angle of 30° with the P.P.. Its nearer extremity is 5′ to the right of the spectator and 5′ within the picture.

PLATE IV.

PROBLEM 4, Fig. I. *An equilateral triangle* X Y Z, *of* 7′ *side, lies upon the* G.P.. *Its nearest corner* X, *is* 1′ *to the left, and* 4′ *within the picture. One side vanishes to the left at an angle of* 45° *with the* P.P..

THIS triangle (see Definition 12) has three equal angles of 60°, and three equal sides of 7′. One of the sides is to vanish to the left at 45°. An equilateral triangle is shown at Fig. II.

Find the nearest corner X by the process employed in Problem I.

Find the V.P., and M.P., and measure the side X Y which vanishes to the left, by Problem 3.

We should here remark that V.P. 45°, coincides with the P.D.; so that besides being a M.P. for lines at right angles to the P.P. (see Proposition III.) the P.D. is a V.P. for lines at 45° with the P.P..

Now we have one side X Y of the triangle. But how are we to find the V.P. of the next side? As the triangle has three equal angles of 60° *the next side will be at an angle of* 60° *with* Y X. Therefore, find the V.P. of the second side by drawing a *second Vanishing Parallel at an angle of* 60° *with Vanishing Parallel* 45°. This will give V.P. 2nd side.

Join X to V.P. 2nd side.

Find M.P. 2nd side (see Proposition VII.).

Transfer X to G.L. (Proposition V.) from M.P. 2nd side in X 2nd side.

Measure 7′ from X 2nd side, in 7′ 2nd side.

Join 7′ 2nd side to M.P. 2nd side, giving X Z, the 2nd side of the triangle.

Join Z to Y and the triangle is complete.

The following exercise should now be worked:—

EXERCISE 4. An equilateral triangle of 10′ side has its nearest corner directly opposite the spectator and touching the P.P.. The triangle is upon the G.P.. One side recedes from the P.P. at an angle of 60° *to the left.* In working this exercise it will be found that the M.P. of one side of the triangle is the V.P. of another side.

PROBLEM 5, Fig. III. *A Square of* 8′ *side, lies upon the* G.P. *with one side vanishing to the right at* 50° *with the* P.P.. *The nearest corner is* 3′ *to the left and* 5′ *within the picture.*

A SQUARE is a figure having four equal sides. See Fig. IV. Its *opposite* sides are *parallel* to each other. Its *adjacent* sides are at *right angles* with each other. A line joining opposite corners is a *diagonal*, as X Z. A line passing through the centre parallel to two sides, and meeting two other sides, is a *diameter*, as D M. The *diagonals* make angles of 45° with the *sides* and *diameters* of the square.

Find the nearest corner X by Problem I.

Find V.P. 50°, and M.P. 50°, and measure the first side, X Y by the process employed for X Z Problem 3.

As the next side of the square is at *right angles* to X Y. Find V.P.R.A. (Vanishing Point at right angles) *by drawing a second Vanishing Parallel at right angles to Vanishing Parallel* 50°.

Join X to V.P.R.A..

Find M.P.R.A. (Proposition VII.).

Transfer X to G.L. in X R A (Proposition V.).

Measure 8′ from X R A in 8′ R A.

Join 8′ R A to M.P.R.A. giving X W, the second side of the square.

Proposition VIII. LINES WHICH ARE PARALLEL TO EACH OTHER VANISH TO THE SAME POINT.

Side W Z is parallel to X Y.

Side Y Z is parallel to X W:—Therefore join W to V.P. 50°, and Y to V.P.R.A. and the two lines will meet in Z, completing the figure. It will here be noted that lines W Z and Y Z measure each other.

Work out the following exercise:—

EXERCISE 5. A rectangle, 15′ long, and 10′ wide, lies upon the G.P.. Its short sides vanish to the right at an angle of 40° with the P.P.. The nearest corner is 3′ to the right of the spectator, and 6′ within the picture. What is the angle made by a diagonal of the rectangle with the P.P.?

Fig. I.

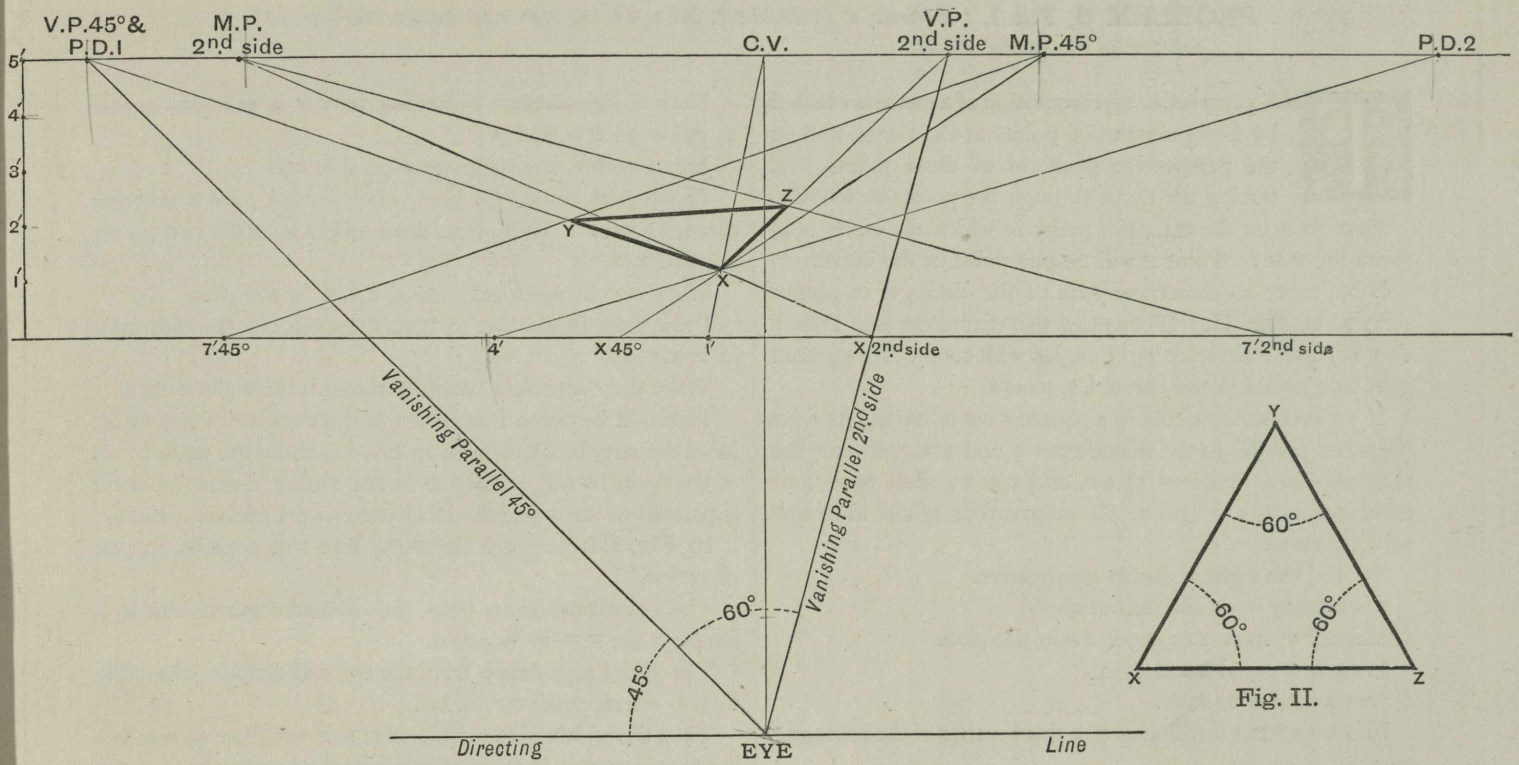


Fig. II.

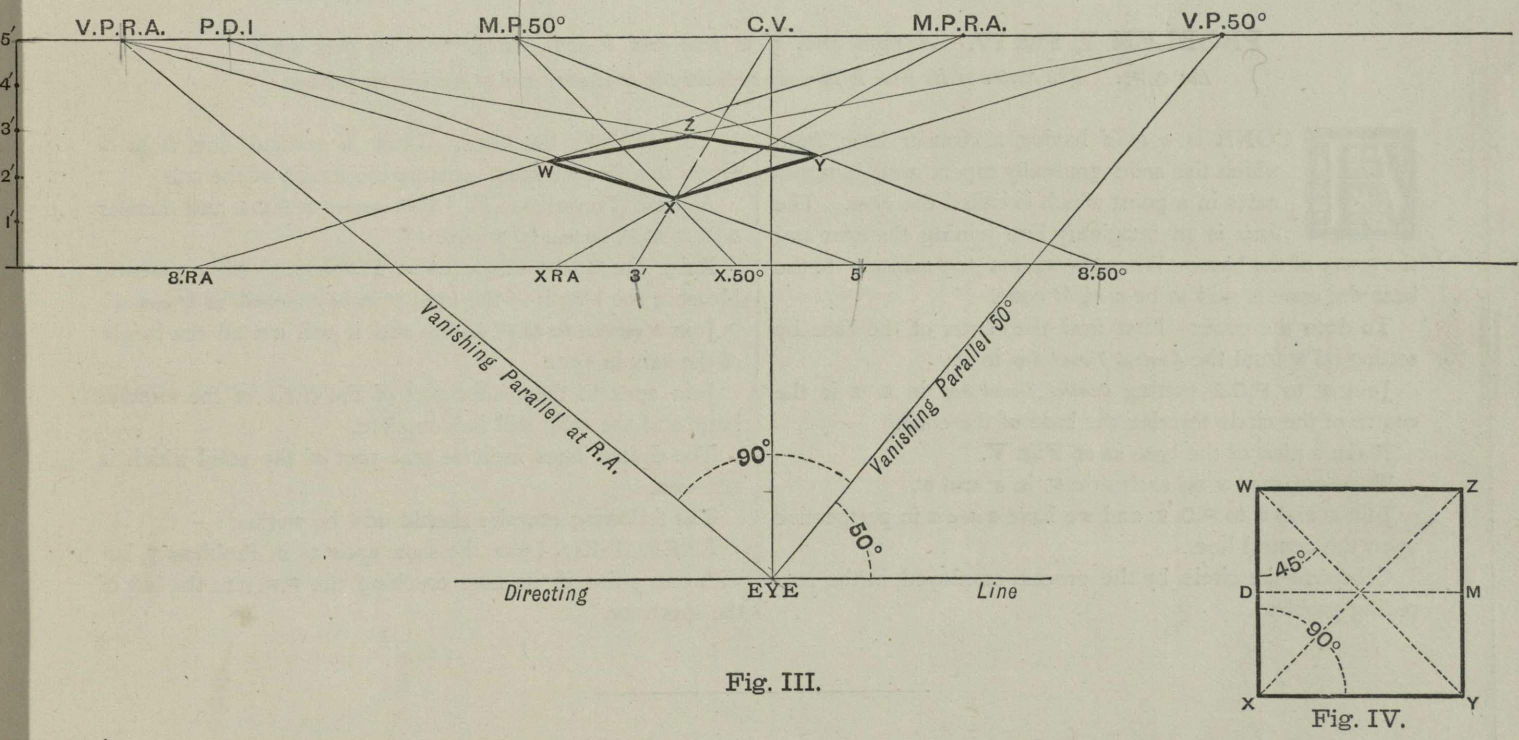


Fig. III.

Fig. IV.

PLATE V.

PROBLEM 6, Fig. I. *A circle,* 8′ *in diameter, lies upon the* G.P. *and touches the* P.P. *at a point* 2′ *on the spectator's left.*

THE perspective representation of a curve is effected by fixing a series of points in the curve, finding the perspective positions of these points, and tracing the curve through the points so found.

First fix 2′ on the G.L., the point in which the circle is to touch the P.P.. Point 2′ will be one point in the curve.

Next, make a geometrical *plan* of the circle, of its proper size; as at **Fig. II.** If we draw two diameters (see Definition 11) of the circle, at right angles with each other we shall gain four points in the curve, 1, 2, 3 and 4.

If we enclose the circle in a square A B C D, having its sides (in pairs) parallel to the diameters 1 2, and 3 4, and draw the diagonals (see Problem 5) A D, and C B, we shall have four other points 5, 6, 7 and 8; *the intersections of the diagonals with the curve.*

To find the eight points in perspective.

2 is already upon the G.L. at 2′.

Make 2′ A′ upon G.L. equal 2 A in the plan.

Make 2′ B′ equal 2 B in plan.

Join A′ and B′ to C.V..

Join B′ to P.D.1 it will give C′, a third corner of the enclosing square.

Draw C′ D′ parallel to A′ B′ and the enclosing square is complete.

Draw the diagonals C′ B′ and A′ D′ meeting in O, the centre of the square.

Draw a line through O *parallel* to A′ B′, it will give points 3′ and 4′ on C′ A′ and D′ B′.

Join 2′ to C.V. it gives point 1′ on side C′ D′.

In the plan draw two lines, one *through* 7 and 5, another *through* 6 and 8. These lines meet side A B, in the two points marked X, X.

Make 2′ X′, X′, upon G.L., equal 2 X, X, in the plan.

Draw lines from X′, X′, to C.V., they will cut the diagonals in 5′, 6′, 7′, 8′.

Trace the curve very carefully through the eight points.

It should be noted that although the *diameter* of the circle, is—if we may be allowed so to call it—really the *widest* part of the figure, it will not *appear* as the widest; unless the circle is parallel to the P.P., with its centre exactly opposite the eye.

In **Fig. III.** we have the P.P., Eye and a circle, in side elevation.

The *Visual Rays* drawn from the diameter M D to the eye, intersect the P.P. in m and d.

The visual rays drawn from the eye and *touching* the circle in T, T, intersect the P.P. in t, t.

Thus the width of the circle at T, T, is *less* than at M D, but it *appears greater*, because it is nearer the spectator: t t being greater than m d.

Work the following exercise:—

EXERCISE 6. Draw the same circle as in Problem 6, but place its nearest point 3′ to the right, and 6′ within the picture.

PROBLEM 7, Fig. IV. *A right cone, of* 6′ *base and* 8′ *axis, stands with its base upon the* G.P.. *The centre of its base is directly opposite the spectator, and* 5′ *within the picture.*

A CONE is a *solid* having a circular base, from which the solid gradually tapers until it terminates in a point which is called the *apex.* The *axis* is an imaginary line joining the *apex* and the *centre* of the base. When the *axis* is *perpendicular* to the base the cone is said to be a *right* cone.

To draw the cone:—First find the centre of the base by setting off 5′ from the *Central Visual Ray* in 5′.

Join 5′ to P.D.2 cutting *Central Visual Ray* in 5. 5 is the centre of the circle forming the base of the cone.

Make a plan of the base as at **Fig. V.**

Then measure 3′ on each side 5′, in 3′ and 6′.

Join 6′ and 3′ to P.D. 2, and we have 6 and 3 in perspective upon the central line.

Complete the circle by the process employed in the preceding problem.

To complete the cone. Draw a vertical line from 5. Upon this line we must measure the length of the axis.

Assume (Proposition IV.) M.P. vertical at P.D.1. and transfer 5 from M.P. vertical in 5′ vertical.

Raise the Line of Heights (as in Problem 2) from 5 vertical. Measure the length of the axis, 8′ from 5 vertical, in 8′ vertical.

Join 8′ vertical to M.P. vertical and it will cut off the height of the axis in *APEX.*

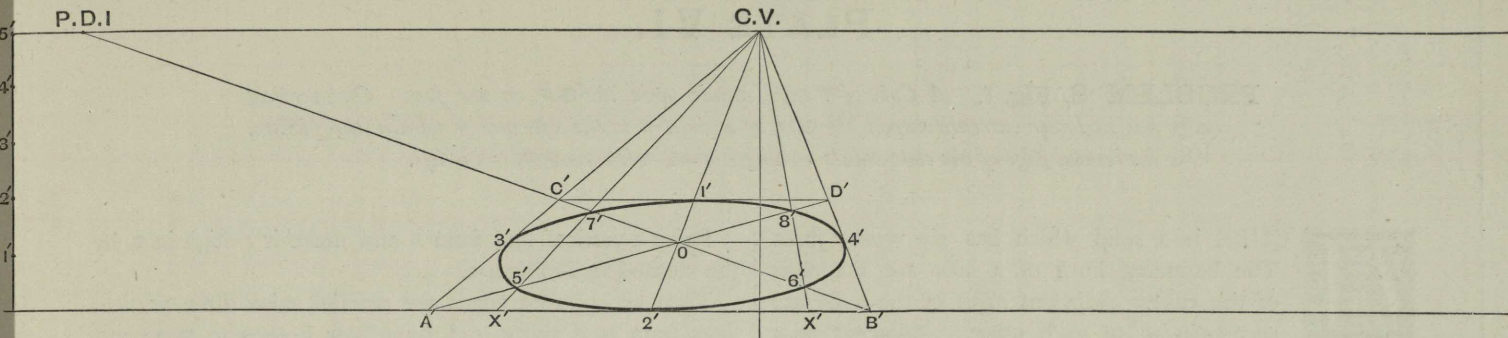
Join apex to the *widest* part of the curve of the circular base, and the solid will be complete.

The dotted lines indicate that part of the solid which is not seen.

The following exercise should now be worked:—

EXERCISE 7. Draw the same cone as in Problem 7, but with one point in its base touching the P.P. 1′ to the left of the spectator.

Fig. I.



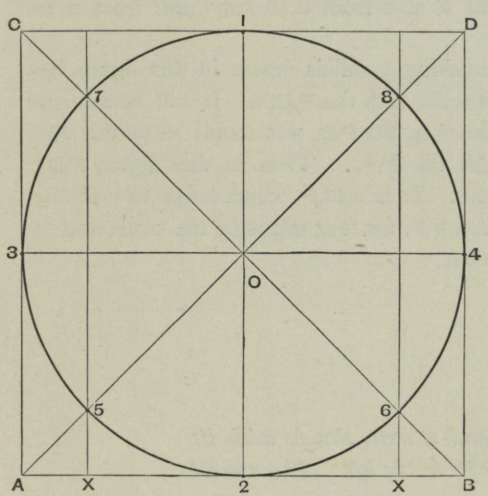


Fig. II.

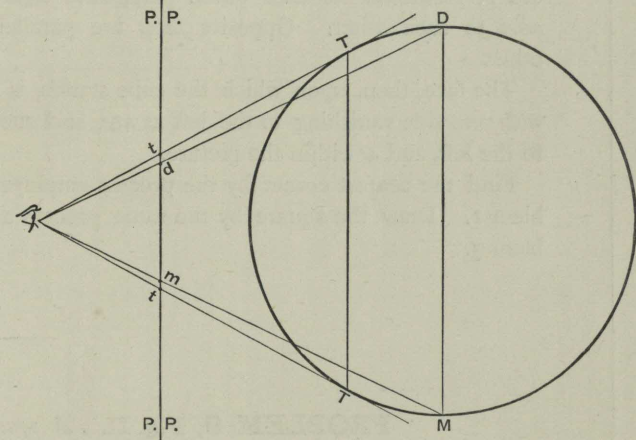


Fig. III.

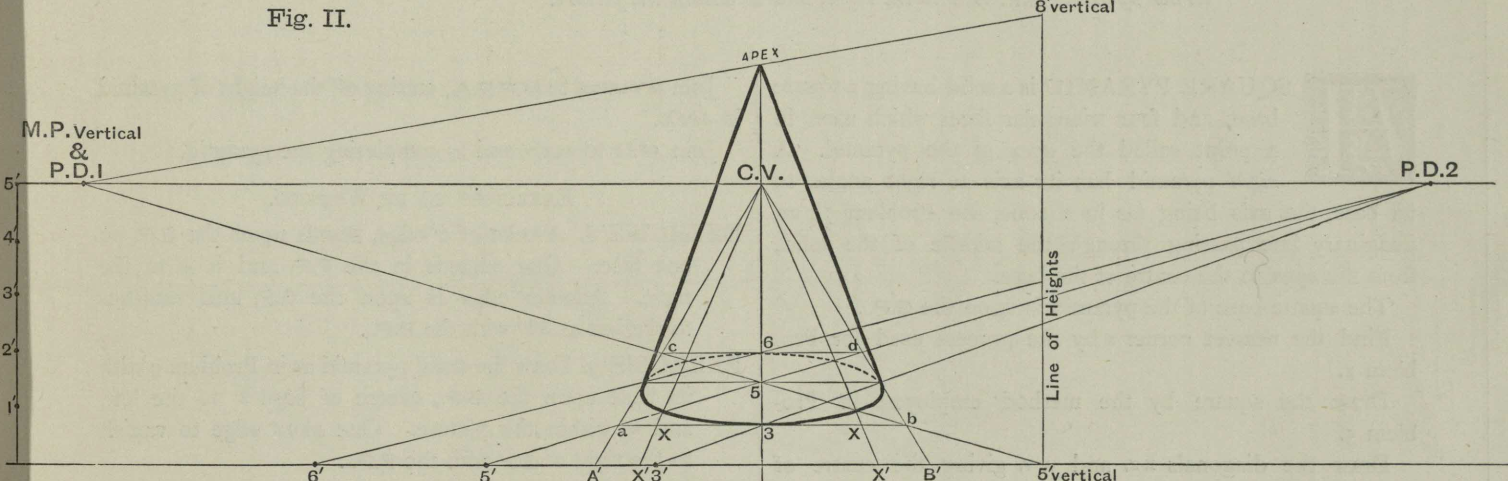


Fig. IV.

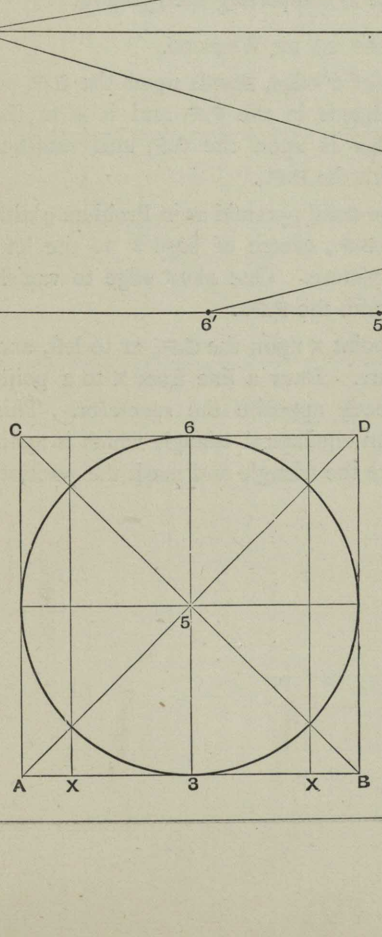


Fig. V.

PLATE VI.

PROBLEM 8, Fig. I. *A Cube of 7′ edge, stands upon the* G.P. *on one face. One vertical edge has its lower extremity upon the* G.P. *at a point 2′ to the left, and 4′ within the picture. One horizontal edge of the cube makes an angle of 45° with the* P.P. *to the left.*

A CUBE is a solid which has *six square faces.* The bounding lines of a face are the *edges* of the cube. Adjacent *edges* of the cube are *perpendicular* to each other. Adjacent *faces* are *perpendicular* to each other. Opposite *edges* are *parallel* to each other. Opposite *faces* are parallel to each other.

The face, then, upon which the cube stands, is a *square;* with one side vanishing to the left at 45°, and one corner 2′ to the left, and 4′ within the picture.

Find the nearest corner by the process employed in Problem 1. Draw the square by the same process as in Problem 5.

Raise a vertical line from X and make it 7′ high in B, by the method in Problem 2.

Then as opposite edges are parallel, raise three vertical lines from W, Z, and Y, and draw lines from B to P.D.1 and P.D.2, giving C and D, also from C to P.D.1 and from D to P.D.2, giving E.

There are no vanishing parallels drawn in this figure because the V.P.'s coincide with the P.D.'s. It will be remembered that in Problem 4, the P.D. was found to be the V.P. for lines at 45° with the P.P.. Thus, in this figure, P.D.1 performs *three* offices. It is a M.P. which helps to find the nearest corner, it is a V.P. for four edges of the cube, and it is used as M.P. Vertical.

PROBLEM 9, Fig. II. *A square, right pyramid, of 6′ base, and 8′ axis, stands with its base upon the* G.P.. *One edge of its base is at 40° with the* P.P. *to the left. The nearest corner upon the* G.P. *is 3′ to the right, and 3′ within the picture.*

A SQUARE PYRAMID is a solid having a square base, and four triangular faces which meet in a point called the *apex* of the pyramid. A *right* pyramid has its axis at right angles to its base, the axis being (as in a cone, see Problem 7), an imaginary line passing through the middle of the solid, from the apex to the centre of the base.

The square base of the pyramid is upon the G.P..

Find the nearest corner X by the process used for Problem 1.

Draw the square by the method employed in Problem 5.

Draw the diagonals XZ, and WY, giving the centre of base C.

From C raise a Vertical line.

Assume M.P. Vertical at M.P.R.A. and transfer C to G.L. in C Vertical.

Raise Line of heights at C Vertical. Measure 8′ from C Vertical in 8′ Vertical.

Join 8′ Vertical to M.P.R.A., cutting off the height of pyramid in APEX.

Join APEX to W, X, Y, and Z; completing the pyramid.

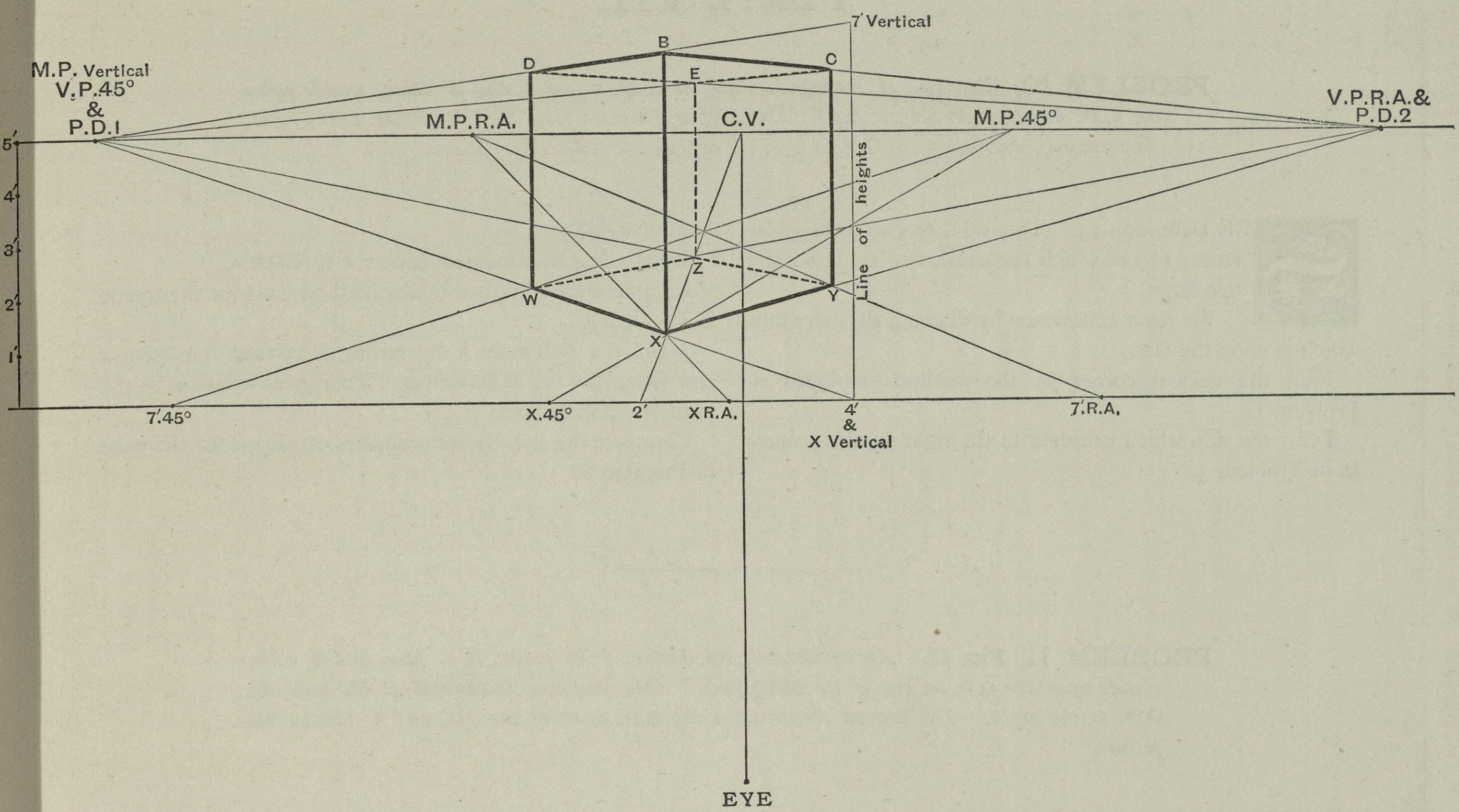
EXERCISES TO BE WORKED.

EXERCISE 8. A cube of 5′ edge, stands upon the G.P. on one face. One edge is in the P.P. and is 4′ to the right. Another edge is upon the G.P. and vanishes to the left at 35° with the P.P..

EXERCISE 9. Draw the same pyramid as in Problem 9 with its base upon the G.P., centre of base 2′ to the left, and 6′ within the picture. One *short* edge to vanish to the right at 30° with the P.P..

EXERCISE 10. Find point X upon the G.P., 12′ to left, and 12′ within the picture. Draw a line from X to a point upon the G.L. directly opposite the spectator. This line is one side of an equilateral triangle which is upon the G.P.. Complete the triangle and mark the position of its centre.

Fig. I.


7' Vertical
B
D
C
E
M.P. Vertical
V.P.45°
&
P.D.1
M.P.R.A.
C.V.
M.P.45°
V.P.R.A.&
P.D.2
5'
4'
3'
2'
1'
Line of heights
Z
W
Y
X
7'.45°
X.45°
2'
X R.A.
4'
&
X Vertical
7'.R.A.
EYE

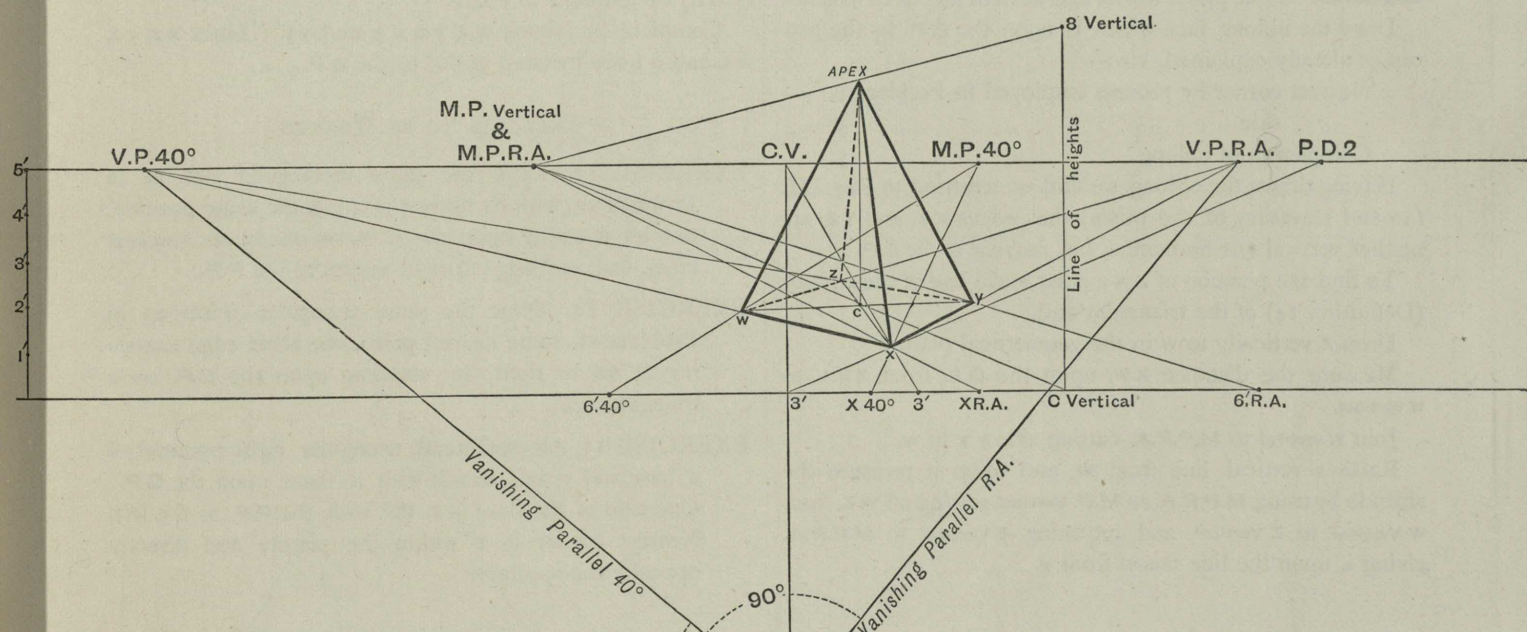

8 Vertical
APEX
M.P. Vertical
&
M.P.R.A.
V.P.40°
C.V.
M.P.40°
V.P.R.A.
P.D.2
5'
4'
3'
2'
1'
Line of heights
Z
C
W
Y
X
6'.40°
3'
X 40°
3'
XR.A.
C Vertical
6'.R.A.
Vanishing Parallel 40°
Vanishing Parallel R.A.
90°
40°
Directing
EYE
Line

Fig. II.

Plate VII.

PROBLEM 10, Fig. I. *A rectangular slab* 12′ *long,* 6′ *wide, and* 3′ *thick, stands upon the* G.P. *on one of its largest faces. One long edge recedes from the* P.P. *at* 50° *to the right. The nearest corner upon the* G.P. *is* 5′ *to the left, and* 4′ *within the picture.*

SEE Definition 13. This solid has six *rectangular* faces; two of which are small, two medium, and two large.

We must commence by drawing the *large* face which is upon the G.P..

Find the nearest corner by the method employed in Problem 1.

Draw the side which vanishes to the right by the process as in Problem 3.

Find V.P.R.A..

Draw a line from nearest corner X to V.P.R.A..

Complete the rectangle by the method used for the square in Problem 5.

Make the slab 3′ thick by raising a vertical line from X and using C.V. as M.P. Vertical. This gives X Vertical in the same position as point 5′.

Complete the slab by the processes employed for the cube in Problem 8.

PROBLEM 11, Fig. II. *An equilateral, triangular, right prism, of* 5′ *base, and* 9′ *axis, stands upon the* G.P. *on one of its oblong faces. One long edge is inclined at* 45° *with the* P.P. *to the right. The nearest corner upon the* G.P. *is* 3′ *to the left, and* 4′ *within the picture.*

A TRIANGULAR PRISM is a solid having two triangular ends and three oblong faces. The axis is a line joining the centres of the two ends. A right prism has its axis at right angles to its ends.

Draw the oblong face which is upon the G.P. by the processes already explained, viz.:—

Nearest corner by process employed in Problem 1.
" side " " " 3.
Completion of oblong " " 5.

Having drawn the oblong, we find on referring to **Fig. III.** (an end elevation of the prism) that edges X Z, and Y Z, are neither vertical nor horizontal, but *inclined to the* G.P..

To find the position of Z we must make use of the *altitude* (Definition 14) of the triangular end.

Drop Z vertically to W in the geometrical plan.

Measure the distance X W, upon the G.L. from X R.A. to W vertical.

Join W vertical to M.P.R.A. cutting side y x in w.

Raise a vertical line from w, and upon it measure the altitude by using M.P.R.A. as M.P. Vertical, setting off W Z, from W Vertical to Z Vertical, and returning Z Vertical to M.P.R.A. giving z, upon the line raised from w.

Transfer w, to the *distant, short* edge v u, in l, by a line going to P.D.2 (see Proposition VIII.).

Raise a vertical line from l, and measure l t, (Proposition VIII.) by joining z to P.D.2.

Complete by joining x z, y z, u t, and v t. Lines x z, y z, v t, and u t, are inclined at 60° to the G.P..

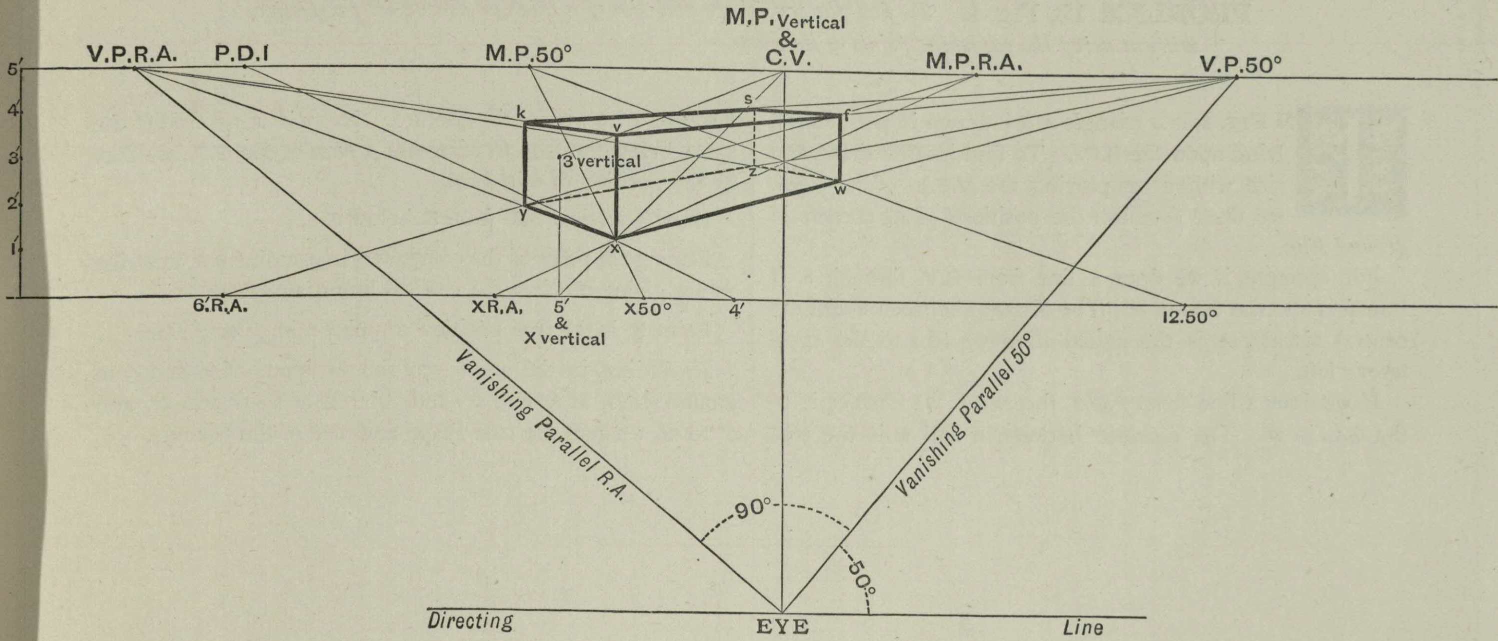
Exercises to be Worked.

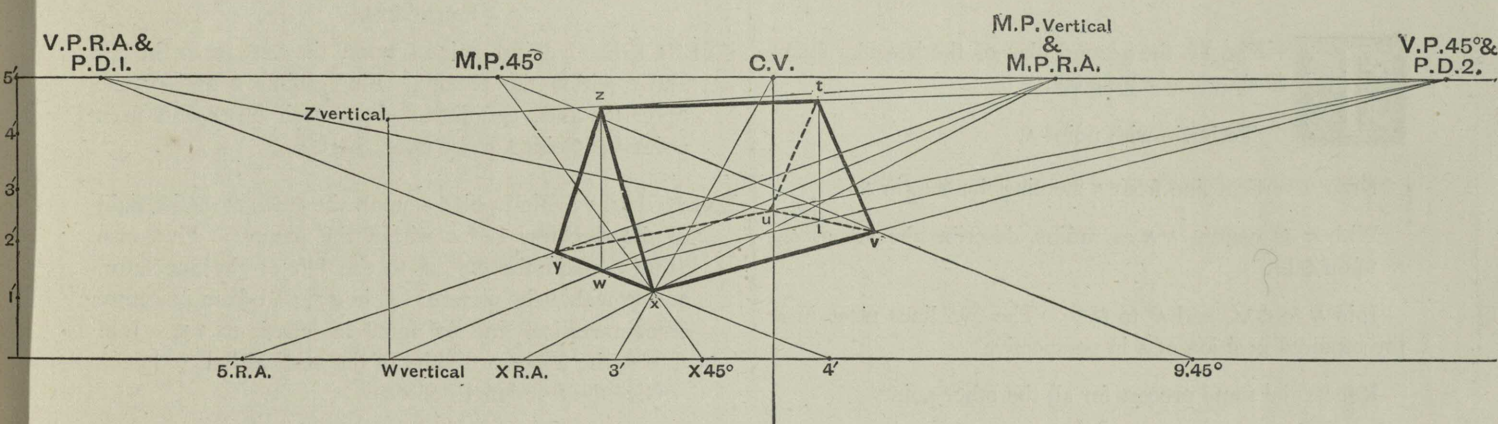
EXERCISE 11. Draw the same rectangular slab as in Problem 10, with its nearest point in the same position, but let it stand upon the G.P., on one of its smallest faces, with one edge at right angles to the P.P..

EXERCISE 12. Draw the same triangular prism as in Problem 11, same nearest point, one short edge vanishing at 45° to right, but standing upon the G.P. on a triangular end.

EXERCISE 13. An equilateral, triangular, right pyramid of 6′ base and 7′ axis stands with its base upon the G.P.. One side of the base is at 25° with the P.P. to the left. Nearest corner is 3′ within the picture and directly opposite the spectator.

Fig. I.





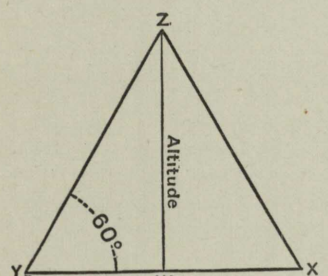


Fig. III.

Fig. II.

PLATE VIII.

PROBLEM 12, Fig. I. *To find the true shape and size of a triangle drawn in perspective, without using the vanishing points of its sides.*

IN **Fig. I.** is a triangle a, b, c, drawn in perspective, lying upon the G.P.. To find its true shape and size without employing the V.P.'s. of its sides, we must establish the positions of its corners in *ground plan*.

For example, if we draw a line from C.V. through b, it transfers b to the G.L. in B. The distance between B and the central visual ray is the actual distance of b to the spectator's left.

If we draw a line from P.D.2. through b, we transfer it to the G.L. in B′. The distance between B and B′ is the true distance of b within the picture. Now, if we consider the space below the G.L. to represent a *plan* of the G.P., we may fix the position of b in plan.

Draw a vertical line from B, below G.L..

Placing the point of the compasses in B, radius B B′, describe an arc giving B″ upon the vertical line drawn from B.

Point B″ is the true position of point b in *ground plan.*

By finding, in the same way the positions of a, and c, in ground plan, at A″ and C″, and joining A″ to B″ and C″, and C″ to B″, we gain the true shape and size of the triangle.

PROBLEM 13, Fig. II. *To draw an irregular figure in perspective, the ground plan being given.*

IN **Fig. II.** the ground plan of the irregular figure is shown at A,B,C,D,G,F.

To begin with point A.

Draw a vertical line from A meeting the G.L. in A′.

With A′ as centre, A′ A as radius, describe the arc giving A″ upon G.L..

Join A′ to C.V., and A″ to P.D. The two lines meet in a, the required position of A in perspective.

Repeat the same process for all the other points.

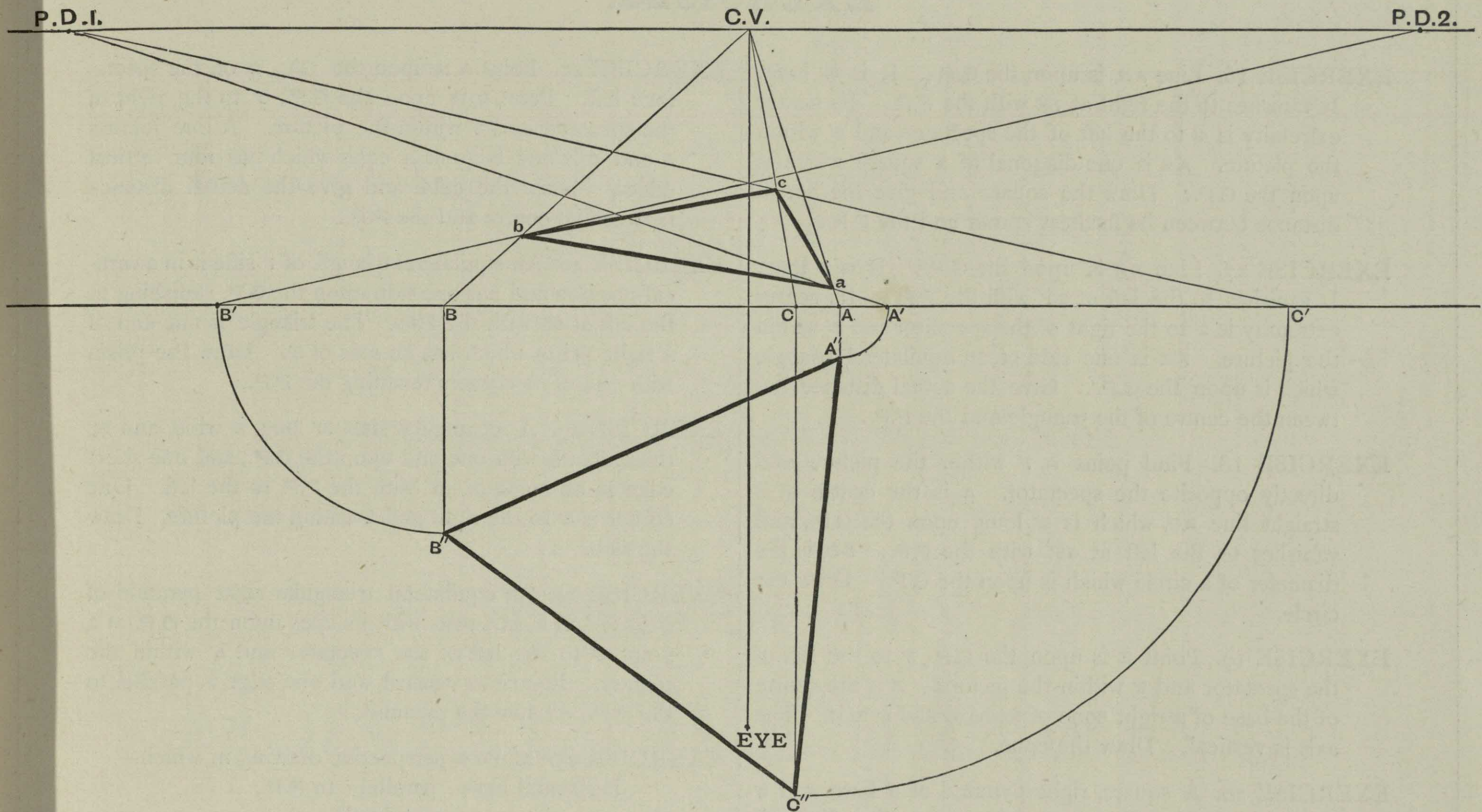
EXERCISES.

EXERCISE 14. Find point A, upon the G.P., 5′ to the left and 5′ within the picture. Find points B and C also upon the G.P., equidistant from each other and from point A. Point B to be upon the G.L..

EXERCISE 15. Find point X upon the G.P., 3′ to the right of the spectator and 4′ within the picture. Find also point Z upon the G.P. 3′ to the left of the spectator, and 10′ within the picture. Join Z X, and from its centre erect a vertical line H K equal in length to Z X. Join X to K, the upper extremity of the vertical line.

Give the true length of X K.

Fig. I.


P.D.I.
C.V.
P.D.2.
c
b
a
B′
B
C
A
A′
C′
A″
B″
EYE
C″

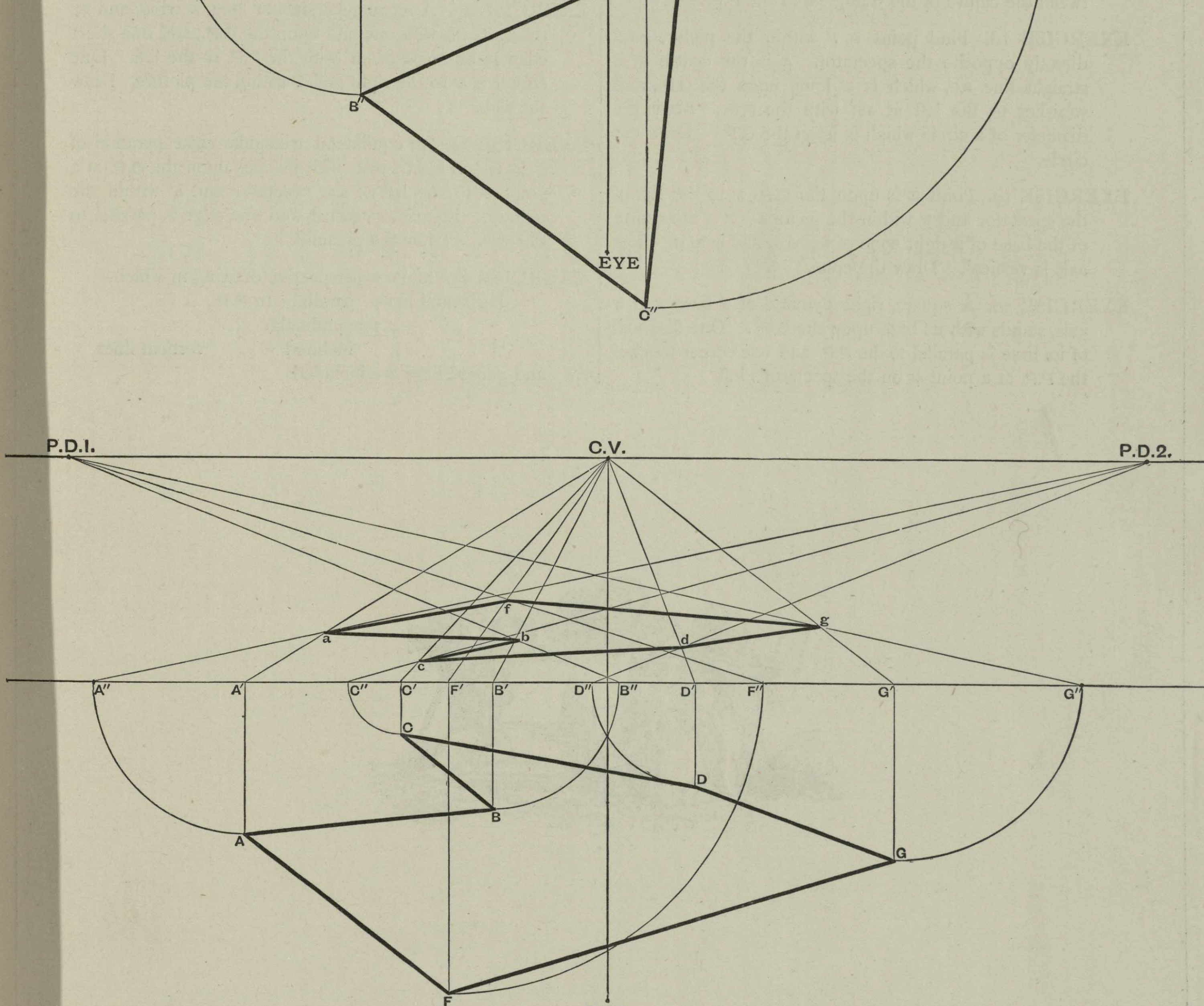

P.D.I.
C.V.
P.D.2.
f
g
a
b
d
c
A″
A′
C″
C′
F′
B′
D″
B″
D′
F″
G′
G″
C
D
B
A
G
F
EYE

Fig. II.

EXERCISES.

EXERCISE 16. Line AB, is upon the G.P.. It is 10′ long. It vanishes to the right at 70° with the P.P.. Its nearer extremity is 4′ to the left of the spectator and 6′ within the picture. AB is one diagonal of a square which is upon the G.P.. Draw the square and give the actual distance between its furthest corner and the P.P..

EXERCISE 17. Line XZ is upon the G.P.. It is 9′ long. It vanishes to the left at 50° with the P.P.. Its nearer extremity is 2′ to the right of the spectator and 3′ within the picture. XZ is one side of an equilateral triangle which is upon the G.P.. Give the actual distance between the centre of the triangle and the P.P..

EXERCISE 18. Find point A, 7′ within the picture and directly opposite the spectator. A is the centre of a straight line BC, which is 9′ long, upon the G.P., and vanishes to the left at 45° with the P.P.. BC, is the diameter of a circle which is upon the G.P.. Draw the circle.

EXERCISE 19. Point Z is upon the G.P., 3′ to the left of the spectator and 9′ within the picture. Z is the centre of the base of a right cone of 10′ base and 5′ axis. The axis is vertical. Draw the cone.

EXERCISE 20. A square, right pyramid of 6′ base and 8′ axis, stands with its base upon the G.P. One diagonal of its base is parallel to the P.P and one corner touches the P.P. at a point 4′ on the spectator's left.

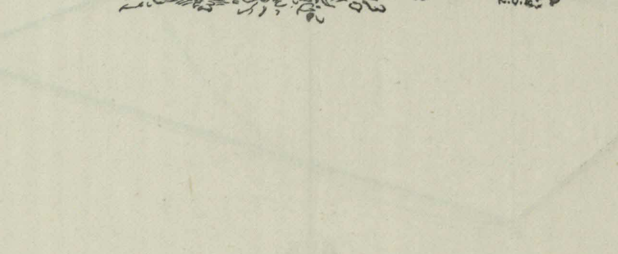
EXERCISE 21. Point A is upon the G.L., 2′ on the spectator's left. Point B is upon the G.P., 3′ to the right of the spectator and 3′ within the picture. A line joining A and B is one edge of a cube which has four vertical edges. Draw the cube and give the actual distance between its centre and the P.P..

EXERCISE 22. An equilateral triangle of 7′ side is in a vertical position and has one side upon the G.P. vanishing to the left at 30° with the P.P.. The triangle is one end of a right prism which has an axis of 9′. Draw the prism with one of its corners touching the P.P..

EXERCISE 23. A rectangular slab 10′ long, 5′ wide, and 2½′ thick, stands with one end upon the G.P., and one short edge at an angle of 40° with the P.P. to the left. One corner is 3′ to the right and 2′ within the picture. Draw the slab.

EXERCISE 24. An equilateral triangular right pyramid of 6′ base and 5′ axis rests with its apex upon the G.P. at a point 2′ to the left of the spectator and 5′ within the picture. Its axis is vertical, and one edge is parallel to the P.P.. Draw the pyramid.

EXERCISE 25. Make a perspective drawing in which—
horizontal lines parallel to P.P.,
" " perpendicular "
" " inclined " vertical lines,
and curved lines are included.



POYNTER'S SOUTH KENSINGTON DRAWING-BOOK.

ELEMENTARY PERSPECTIVE DRAWING.

BY

S. J. CARTLIDGE, F.R.Hist.S.

PART II.

Plate IX.

PROBLEM 14, Fig. I.—*To draw a block of steps* 10′ *long, the end elevation being given.*

IN this, the second division of our work, we still use the same vanishing and measuring points, but the objects to be drawn are rather more difficult than those in Book I.

The end elevation of the steps is shown at **Fig. III.** on the right.

Line A B is to be upon the G.P. and to vanish to the right at an angle of 45° with the P.P., A being directly opposite the spectator and 2′ within the picture.

Find the nearest corner by the process employed in Problem 1, and draw line A B by the method as in Problem 3; as also line a d, giving the *length* of steps, viz. 10′.

The point to be specially noted is the mode of measuring the heights of the *risers* of the steps. They are shown in the elevation at A W, P Q, and R S.

Line Q P produced meets A B in X.

Line S R produced meets A B in Y.

Find the positions of X and Y by making A 2 X 2, equal A X: and B Y 2, equal B Y.

Join X 2 and Y 2 to M.P.2 giving x and y upon line ab.

As lines A W, P Q, R S, and B T, are all contained in the same vertical plane, they will be measured upon the same vertical line of heights.

Using P.D.2 as M.P. Vertical, transfer a to the G.L. in A 2′.

At A 2′ raise line of heights, and measure from A 2′ the heights of the risers in W′, V′, and Z′ equal to the heights of W V and Z in end elevation.

Return W′, V′, and Z′, to P.D.2, meeting vertical lines drawn from a, x, y, and b; in w, p, q, r, s, and t.

Join w, p, q, r, s, and t, to P.D.1.

Complete the steps by means of vertical lines and lines drawn to P.D.2; the first line being a vertical one starting from d.

PROBLEM 15, Fig. II.—*To draw a square slab, having a square pyramid standing with its base upon the upper face of the slab. The nearest corner of the slab is* 3′ *to the right of the spectator and* 3′ *within the picture. The base of the slab is upon the* G.P. *and one edge vanishes to the right at an angle of* 30° *with the* P.P..

THE plan of the two solids is shown at **Fig. IV.** and the elevation at **Fig. V.** From these two drawings all measurements may be taken.

The slab will be easily drawn by reference to Problem 10.

As the base of the pyramid coincides with the upper face of the slab, the only thing to be fixed is the height of the pyramid.

Find k, the centre of the base of the slab, by drawing the two diagonals ac, and bd.

From k raise a vertical line.

Using M.P.30° as M.P. Vertical, transfer k to the G.L. in K.

From K raise *Line of Heights,* and upon it measure from K the combined heights of both solids in V.

Return V to M.P.30°, it gives v upon the vertical line raised from K.

Join v to w, x, y, and z, completing the pyramid.

Exercises.

EXERCISE 26. Draw the same block of steps as in Problem 14, with the nearest corner touching the P.P. at a point 3′ on the spectator's left. The long edges of the steps to vanish to the right at an angle of 35° with the P.P..

EXERCISE 27. Draw the same solids as in Problem 15, with one vertical edge of the slab, in the P.P. 2′ on the spectator's left. Another edge of the slab to be at an angle of 55° with the P.P. to the right of the spectator.

Fig.I.

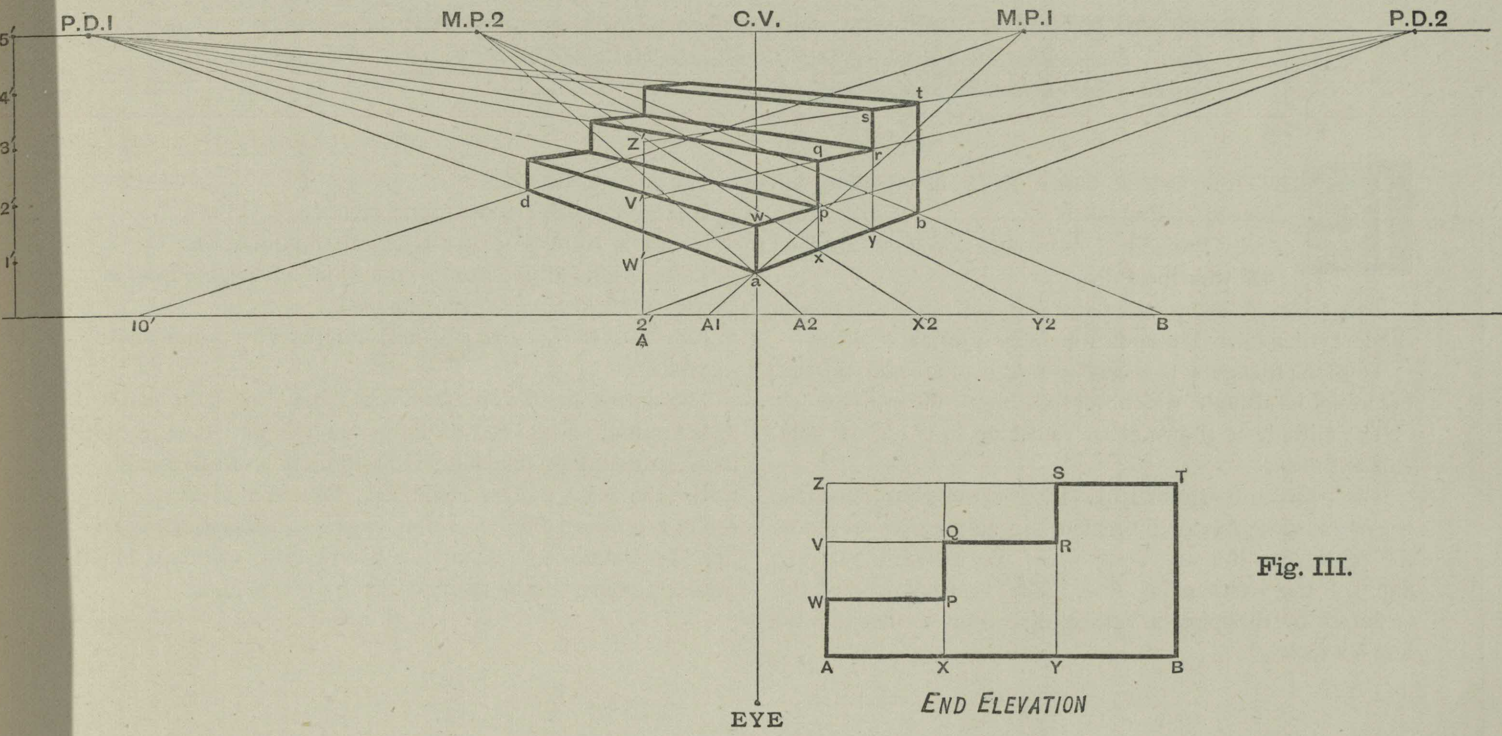


Fig. III.

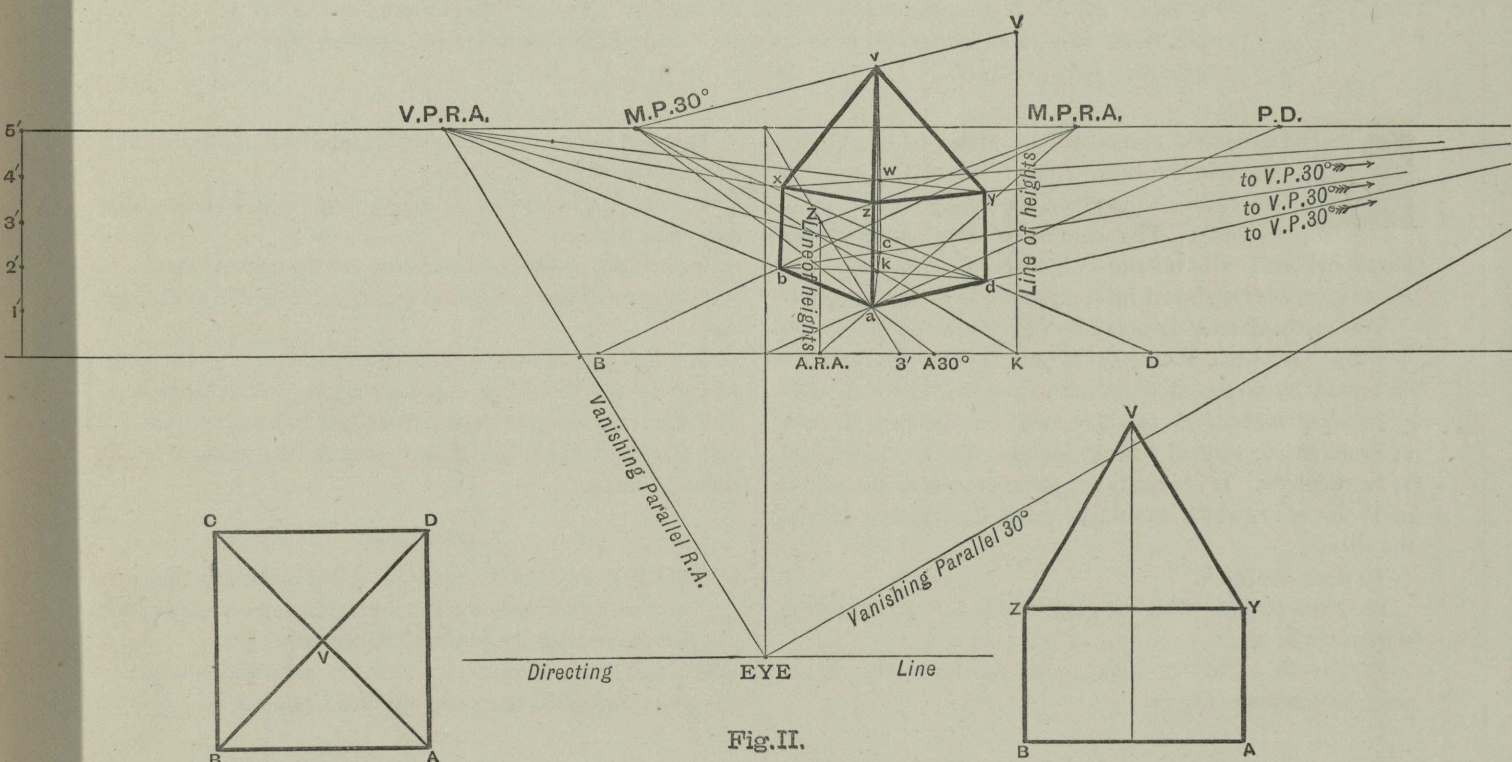


Fig.II.

Fig. IV.

Fig. V.

PLATE X.

PROBLEM 16, Fig. I. *To draw a triangular pyramid lying upon the* G.P. *on one of its long faces. One corner to be* 2′ *to the right of the spectator and* 2′ *within the picture. One edge to vanish to the right at an angle of* 45° *to the* P.P..

FIND the nearest corner X, by the method employed in Problem 1.

By Problem 3 draw edge XY at an angle of 45° with the P.P..

This edge XY being upon the G.P. will form one side of the long triangular face upon which the pyramid will rest.

This long triangular face may be most readily drawn by making use of its altitude, which will start from I, the centre of XY.

The altitude is shown at I a, vanishing to the left at right angles to XY.

The point to be specially remarked is the measurement of the *overhanging base* of the pyramid. In the side elevation, AL represents the side upon which the pyramid is lying, and LP the overhanging *base*. The measurement of this is found by dropping a vertical line from P, meeting the line AY in M.

Transfer I to the G.L. in L′.

Make L′M′ equal LM in side elevation.

Return M′ to M.P.I giving m upon the altitude I a.

Raise a vertical line from m and a line of heights from M′, making M′P′ equal MP in side elevation.

Join P′ to M.P.I giving p, and complete by joining p to a y and x.

The dotted line drawn from a, the apex, to c, the centre of the altitude of the end of the pyramid, is not necessary to the construction of the solid but may help us to thoroughly realize the exact process employed. We have in fact made use of a *vertical plane* containing lines a p, p I, I a and c a. This problem should be well studied as any other pyramid in a similar position will be easily drawn by this method.

PROBLEM 17, Fig. IV. *To draw a hexagonal prism with a stick leaning against it. The prism is* 8′ *long, and stands upon the* G.P. *on one of its rectangular faces. One hexagonal face recedes from the* P.P. *at an angle of* 45° *to the left. The nearest corner upon the* G.P. *is* 3′ *to the left of the spectator, and* 4′ *within the picture. The stick touches one long edge of the prism at a point midway from its hexagonal ends, and is contained in a vertical plane at right angles to its axis.*

THE nearest corner will be found by the process used in Problem 1, and the rectangular face upon which the prism stands, by means of Problem 5. The corners of the nearer vertical hexagonal end are obtained by the four vertical lines e E, A D, B C, and f F, shown in Perspective at e E, a d, b c, and f F.

The inclined stick is measured by means of the vertical line KT. In both Problems 16 and 17 the *inclination* is measured by a vertical line drawn from the *higher extremity of the line inclined to the* G.P., viz. for inclined line PL, in Problem 16, vertical PM, for inclined line ST, in Problem 17, vertical TK. It is exactly the same process as that shown in Problem 11, which should be referred to before drawing the stick.

To draw the stick.

Find the point in which it touches the G.P. by measuring 4′ from 8′2 in 4′2.

Return 4′2 to M.P.2 giving 4 on the long edge of the prism which starts from b.

Through 4 draw a line from P.D.I, producing it indefinitely towards the P.P..

Measure BS from B′ on the right in S. S is found to coincide with B2.

Join B2 & S to M.P.I, it meets edge a b extended, in s.

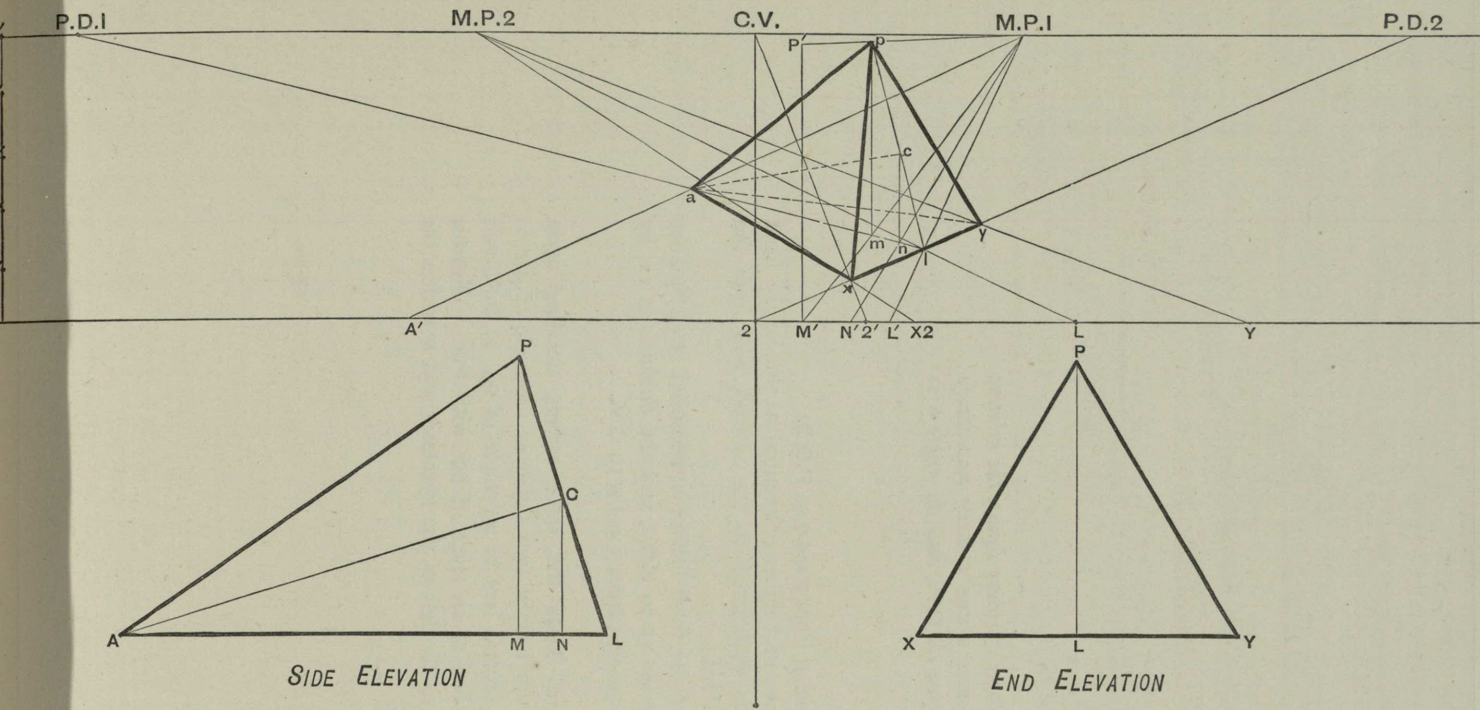
Join s to P.D.2, it meets the line drawn from P.D.I through 4 in s′.

The line of heights for the stick is at K′T′, and the point of contact of the stick is found by a line passing from 4′2 to M.P.2, giving 4, a line drawn from P.D.I through 4 giving k′ and f, and a vertical line from f′ meeting the prism in F′ the point of contact.

EXERCISES.

EXERCISE 28. Draw a pyramid of the same axis and side of base as in Problem 16 and in the same position, but let the base of the pyramid be a *square*.

EXERCISE 29. Repeat Problem 17 substituting a *pentagonal* prism for the hexagonal one.



Side Elevation

Fig. II.

Fig. I.

End Elevation

Fig. III.

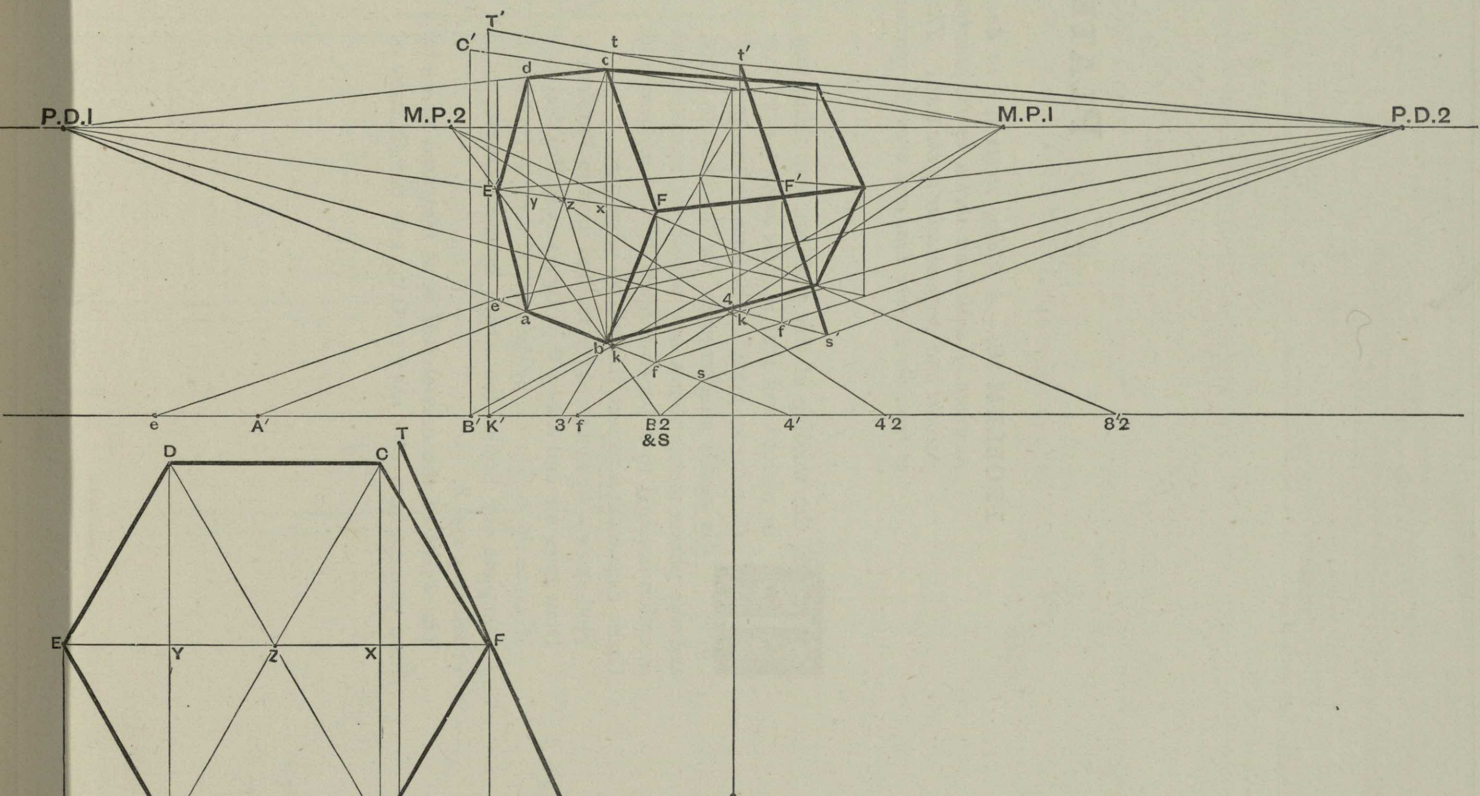


Fig. IV.

End Elevation

Fig. V.

PLATE XI.

PROBLEM 18.—*A folding screen is 7′ in height, and is composed of three leaves, the relative positions of which are shown in the accompanying plan; leaves* B *and* C *being respectively parallel and perpendicular to the* P.P.. *The screen stands upon the* G.P., *and the angle* A *is* 2′ *from the Picture Line and* 3′ *on the spectator's left.*

IN the wording of this problem, one thing may be noticed and that is, "*the angle* A *is* 2′ *from the* PICTURE LINE." The term "PICTURE LINE" is merely another name for the GROUND LINE, and the phrase means that A is 2′ *within the* PICTURE, and attention is called to it because, upon the Government 2nd Grade examination papers, it is a term often used.

Find angle A by means of the method used in Problem 1.

Draw lines A Z and A X by the process as in Problem 3.

Measure the height A a, by the method used in Problem 2.

Complete one leaf of the screen by the processes as in Problems 5 and 8.

The *width* of the central leaf B, is measured at z p by lines passing from Z C V, and V to C.V.; and the *thickness* at p q by lines passing from P and Q to P.D.2.

No further line of heights is required as the height from the first leaf is transferred to the second, and from the second to the third.

The *width* of the third leaf C, is measured at v p by lines passing from P and V2 to P.D.2, and the *thickness* at v w, by means of lines passing from V and W to C.V..

EXERCISE 30. Draw the same screen, standing upon the G.P., the nearest leaf being at an angle of 45° to P.P., the second leaf at an angle of 35° to P.P., and the third leaf at an angle of 60° with P.P.. Nearest angle, 3′ to the left of the spectator and 3′ within the picture.

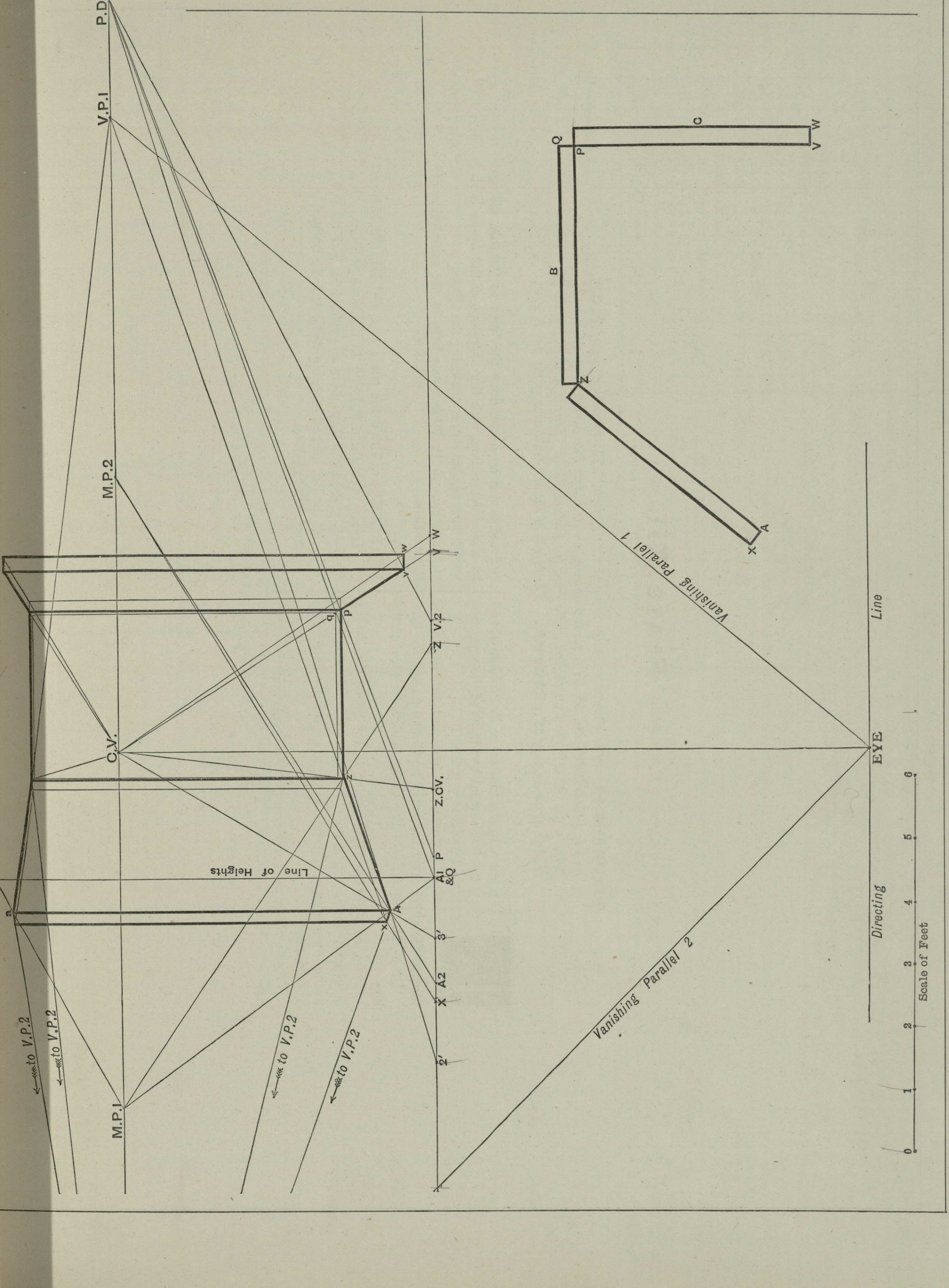

P.D.2
V.P.1
M.P.2
C.V.
M.P.I
Line of Heights
W
V
V.2
Z
Z.CV.
P
A1 &Q
3'
A2
X
2'
←to V.P.2
←to V.P.2
←to V.P.2
←to V.P.2
Vanishing Parallel 1
Vanishing Parallel 2
EYE
Directing Line
Q
P
C
V W
B
Z
X
A
0 1 2 3 4 5 6
Scale of Feet

PLATE XII.

PROBLEM 19.—*The plan of a square box and lid is given. The box stands upon the* G.P., *and is cylindrical inside, the lid is turned upward on its hinges into a vertical position. The point* A (*on the* G.P.) *is* 1' *on the left and* 1' *from the Picture Plane. Line* B, A, *recedes towards the left at an angle of* 35° *with the* P.P.. *The height of the box* (*exclusive of the lid*) *is* 2', 6".

FIND point A by the method as in Problem 1, and draw the box and lid by means of the processes used in Problems 3, 5, and 8.

Having drawn the box and lid, it remains to draw the circle upon the upper horizontal face of the box.

Draw the diagonals of the upper horizontal face of the box.

From C P P (which represents C in the plan) set off the measurements X, X, Z, and Y. Transfer these measurements to the lower edge of the box in 1, 2, 3, 4, and 5, by lines going to M.P. 35°.

From 1, 2, 3, 4, and 5, raise vertical lines meeting the upper near edge in 6, 7, 8, 9, and 10.

From 6, 7, 8, 9, and 10, draw the lines to V.P.R.A. giving by their intersections with the diagonals of the upper horizontal face, square H I J K, enclosing the circle.

A reference to Problem 6 may help in the completion of this problem.

EXERCISE 31. Draw the same box and lid, the nearest corner upon the G.P. being in the same position. One side receding to the right at an angle of 25° with the P.P.: and the lid, instead of being vertical, to be opened in a horizontal position.

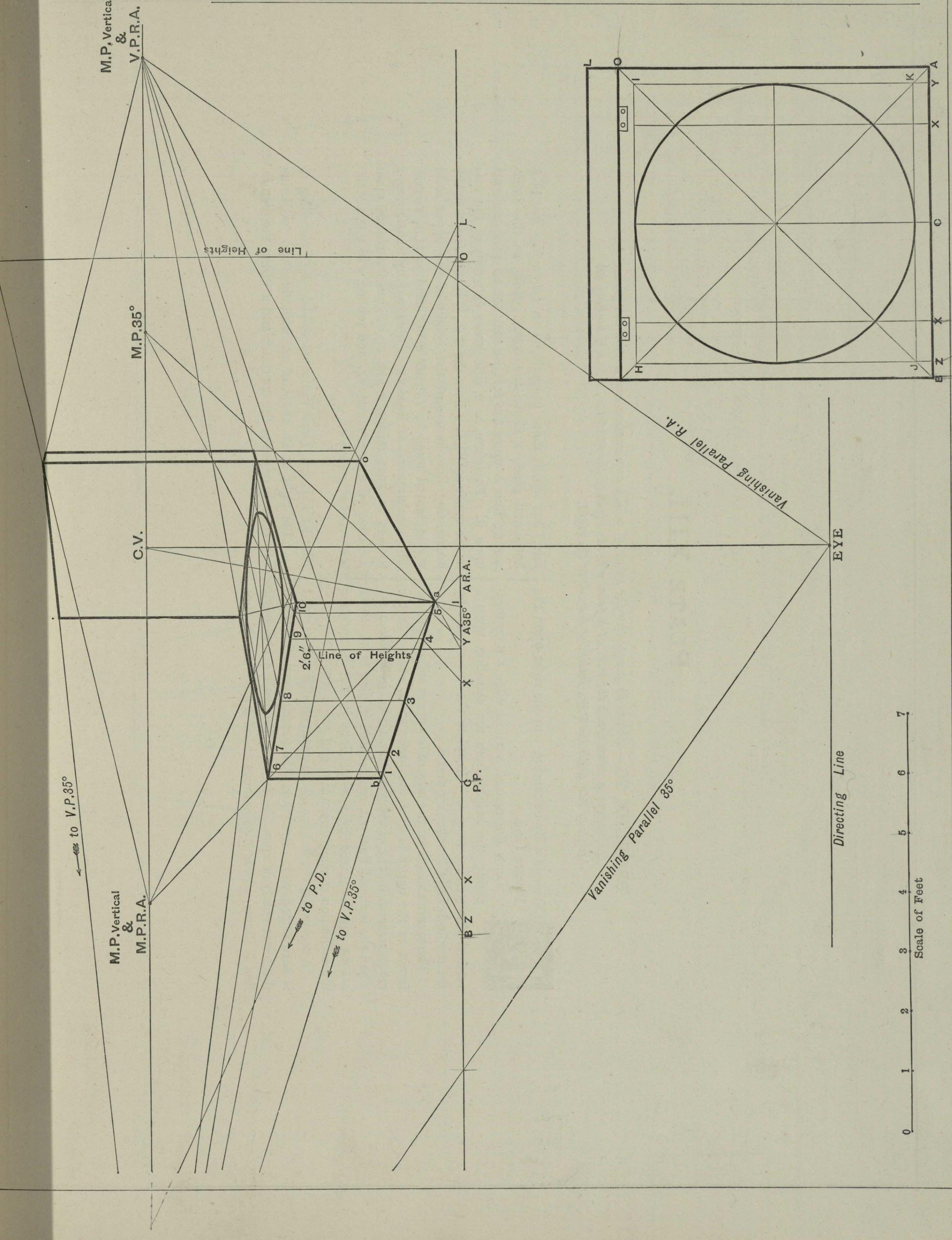

M.P. Vertical & V.P.R.A.
Line of Heights
M.P.35°
C.V.
to V.P.35°
M.P. Vertical & M.P.R.A.
to P.D.
to V.P.35°
2.6"
Line of Heights
A R.A.
Y A35°
X
C P.P.
X
B Z
Vanishing Parallel R.A.
EYE
Vanishing Parallel 35°
Directing Line
Scale of Feet
0 1 2 3 4 5 6 7

PLATE XIII.

PROBLEM 20.—*Draw in perspective the solids of which an elevation and a plan are given. The angle* A *must touch the* P.P. *at a point* 1′ *to the right of the spectator, side* A D *making an angle of* 60° *with the* P.P., *inclining from it to the right.*

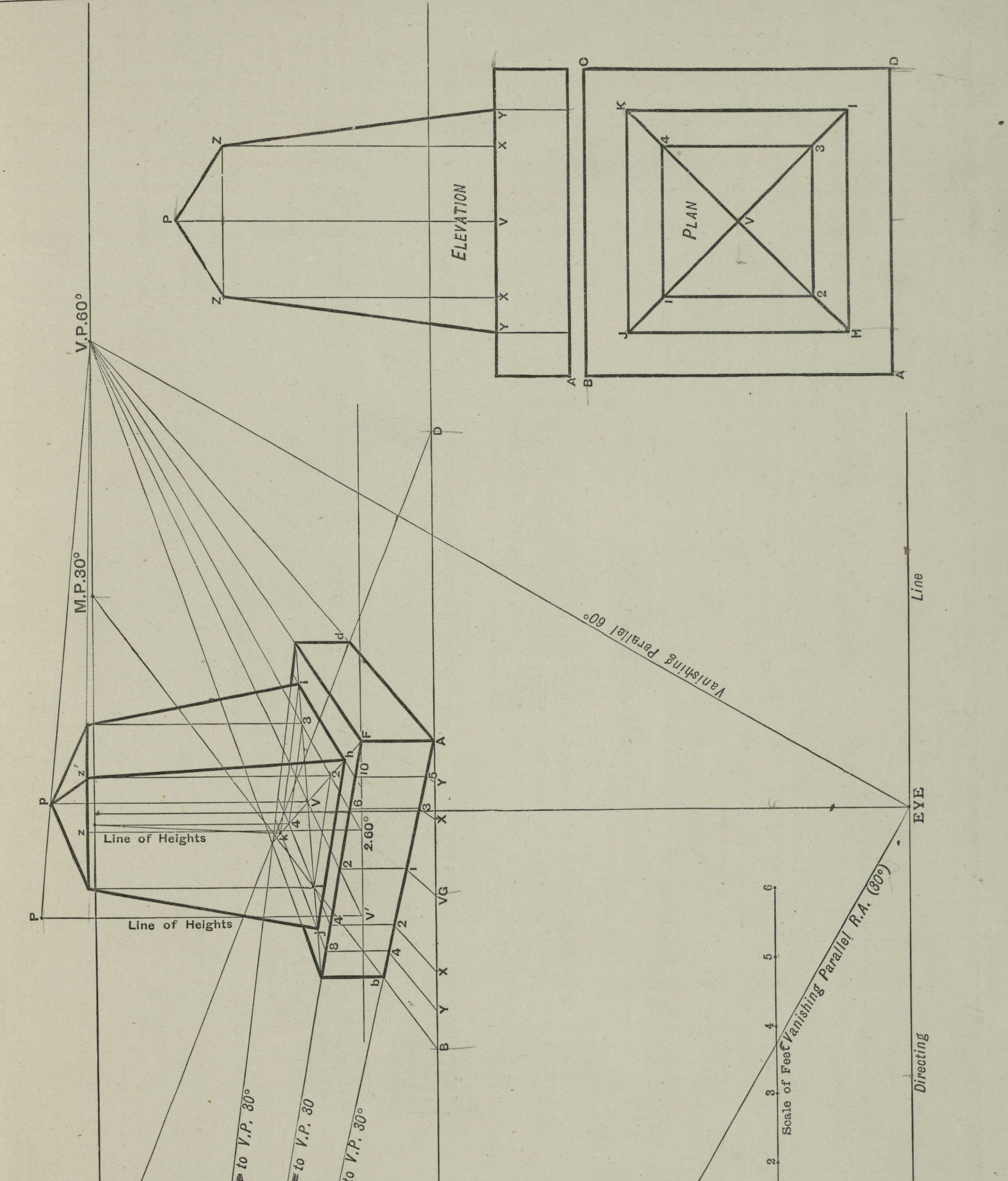
FIX angle A by measuring 1′ to the right upon the Picture Line.

Draw the nearest vertical edge A F by measuring the height from the elevation upon a vertical line starting from A. Complete the lower solid by the methods as in Problems 5 and 8.

The measurements for the base of the upper solid are transferred to the upper face of the lower solid, exactly as the measurements for the circle were transferred in Problem 19. Then the inclined lines Y Z, represented in plan at J 1, H 2, I 3 and K 4, are measured by vertical lines in the perspective representation starting from 1, 2, 3, and 4.

In measuring the heights of the upper solid, a new method is here introduced. Line A F, (the nearest vertical edge of the lower solid) is upon the P.P.. Through F a horizontal line is drawn, which is sometimes called a "floating picture line," but which, more properly, is an *intersecting line* of a horizontal plane 1′ above the G.P.. Now, as this intersecting line is upon the P.P. we may set off measurements upon it. Therefore, as V, (the lower extremity of the axis of the upper solid), is 1′ from the G.P., it is transferred to the intersecting line in V′ and the height of the upper solid V′ P measured upon it, V.P. 60° being used as M.P. Vertical. The measurement of lines X Z is obtained by transferring 2 to P.P. in 2.60°, raising a line of heights from 2.60°, upon it setting off measurement X Z, and joining Z to V.P. 60° the M.P. Vertical, giving Z′.

EXERCISE 32. Draw the same solids. A D, vanishing to the right at an angle of 25° with the P.P.. A being directly opposite the spectator and 3′ within the picture.



V.P.60°
M.P.30°
M.P.60°
ELEVATION
PLAN
Line of Heights
Line of Heights
Vanishing Parallel 60°
Vanishing Parallel R.A. (30°)
EYE
Directing
Line
Scale of Feet
to V.P. 30°
to V.P. 30
to V.P. 30°
2.60°
VG

PLATE XIV.

PROBLEM 21.—*The elevation of a letter is given. It is 3′ thick, stands upon the* G.P., *and its large surfaces are contained in vertical planes which vanish to the left at angles of* 45° *with the* P.P.. *Corner* X, *upon the* G.P. *to be* 4′ *on the spectators' right, and* 4′ *within the picture.*

FIND corner x by the process employed in Problem 1.

Draw lines x 10 and 9 z by the method as in Problem 3.

Draw Y V, the central line of the letter, at y V′, by using M.P.1 as M.P. vertical.

The heights of the points marked 1, 2, and 3, are all determined by means of the same line of heights, viz. Y V.

The invisible portion of the letter is indicated by *thin* lines. This figure may be regarded as a development of Problem 11, to which problem it would be as well to refer.

EXERCISE 33. Draw the letter lying upon the G.P. in a horizontal position, one long edge vanishing to the left at an angle of 15° with the P.P., the nearest corner being directly opposite the spectator and 3′ within the picture.

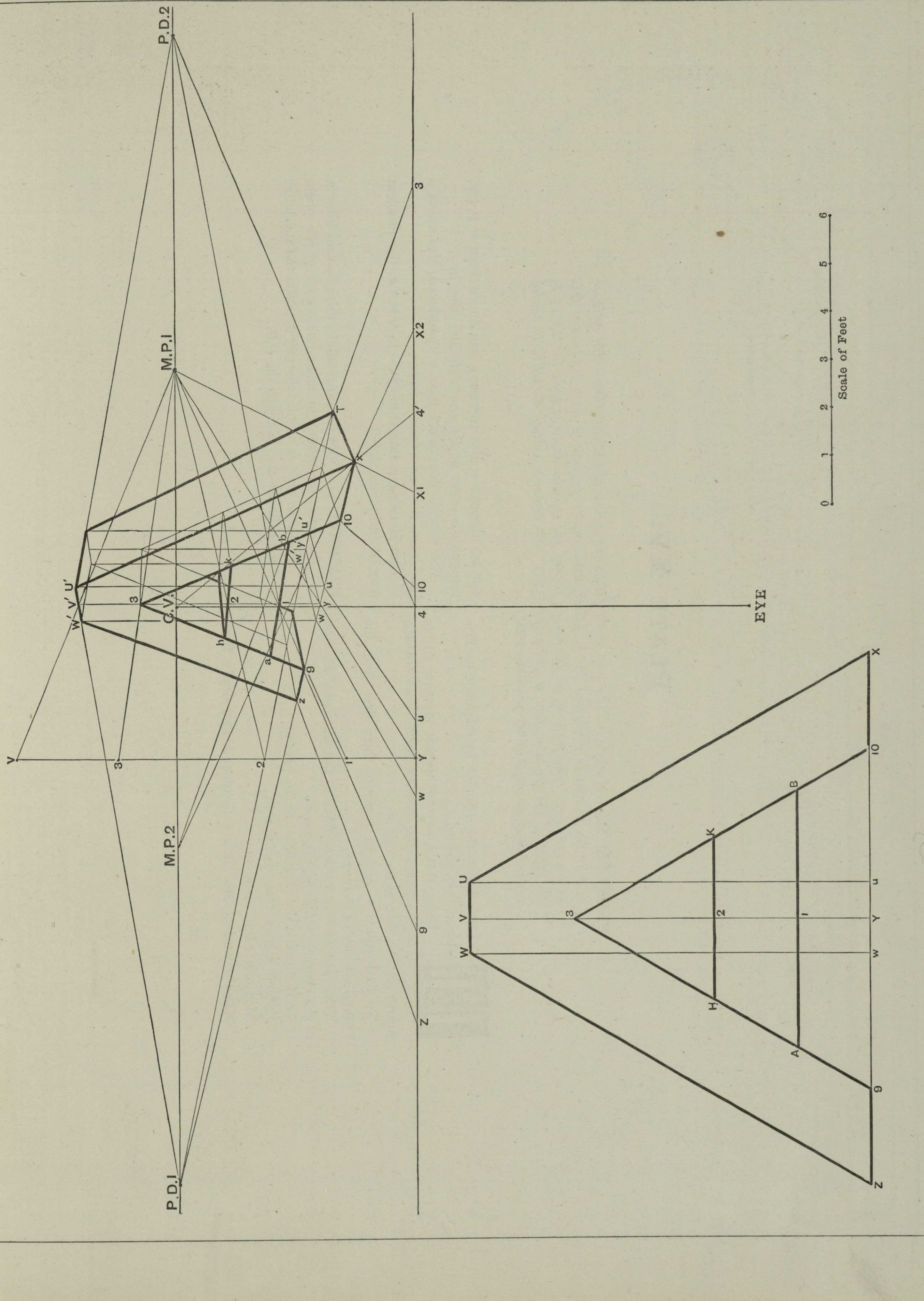

P.D.2
M.P.I
C.V.
M.P.2
P.D.I
EYE
Scale of Feet
0 1 2 3 4 5 6

PLATE XV.

PROBLEM 22.—*A right cylinder and right prism are shown in end elevation. The cylinder is 3' in length and is penetrated by the prism, which latter projects 1½' from the nearer face of the cylinder. The point* A, *on the nearer face of the cylinder is to be upon the* G.P., 1' *to the right of the spectator, and* 5' *from the* P.P.. *The common axis of the solids to vanish to the left at an angle of* 50° *with the* P.P..

HAVING drawn the cylinder by reference to Problems 6, 7 and 19.

Using P.D. 1. as M.P. Vertical, measure all the *heights* upon the *vertical diameter* ab, of the nearer circular face of the cylinder.

To find points F, and E; measure them first upon the ground on line xz in 1 and 2.

Transfer 1 and 2 by vertical lines to the *horizontal diameter* of the nearer circular face in f and e.

To determine the projection of the prism.

Extend line aa, towards the P.P..

Measure 1½' upon it in k.

Raise a vertical line from k to meet a line drawn from V.P. 50° through c; also to meet a line drawn from V.P. 50°, through the *centre* of the nearer circular face, in r.

The vertical line raised from k if produced, also meets a line drawn through d from V.P. 50°.

After drawing a line through r to V.P.R.A. the problem may be easily completed.

EXERCISE 34. Draw the same solids, the axis vanishing in the same direction, but make one corner of the prism touch the P.P. at a point directly opposite the spectator, the cylinder also to touch the P.P..

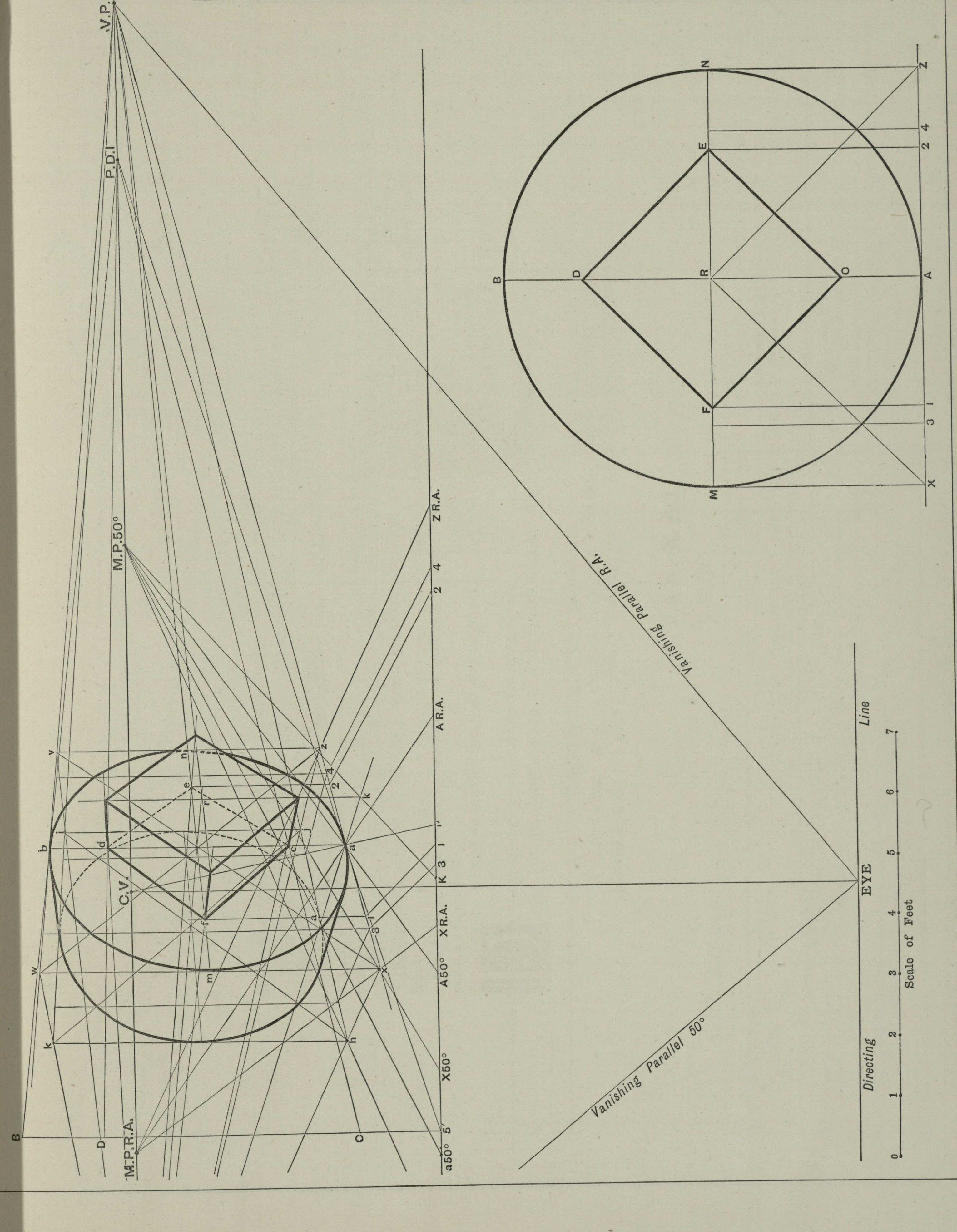

V.P.R.A.
P.D.I
M.P.50°
C.V.
M.P.R.A.
Z R.A.
A R.A.
X R.A.
A50°
X50°
a50°
Vanishing Parallel R.A.
Vanishing Parallel 50°
EYE
Directing Line
Scale of Feet

PLATE XVI.

PROBLEM 23.—*A coffer is shown in plan and elevation. It is to stand upon the* G.P. *on the surface* ABCD. *Line* AB *vanishing to the right at an angle of* 35° *with the* P.P.. *Point* A *is* 1′ *to the left of the spectator and* 3′ *from the* P.P..

DRAW first the rectangle ABCD upon which the coffer stands, shown in the perspective diagram at abcd.

Extend ab towards the P.P., and make av, equal AV in the plan.

Extend ca towards the P.P., and make az equal AZ in the plan.

Draw a line from V.P. 35° through z, also a line from V.P.R.A. through v. These two lines (dotted in the diagram) meet in h.

From h raise a vertical line.

Using V.P.R.A. as M.P. Vertical make the heights m and n equal the heights M and N in the elevation.

To draw the rectangle NOPQ, make use of a floating picture line. (See Problem 20).

Draw this line horizontally through N.

Join n to V.P. 35°.

Transfer n to floating picture line from M.P. 35° in N 35°.

Measure the length NO from N 35° in O 35°.

Join O 35° to M.P. 35°; it gives o.

Repeat the process for the side N, P.

The rest of this figure will be easily understood, noting only that the height AY is measured upon the small line of heights raised at A.R.A. upon the Picture Line, M.P.R.A. being used as M.P. Vertical.

EXERCISE 35. Draw the same coffer, its nearest upper vertical edge touching the P.P. 2′ on the spectator's right, and its vertical surfaces making equal angles with the P.P..

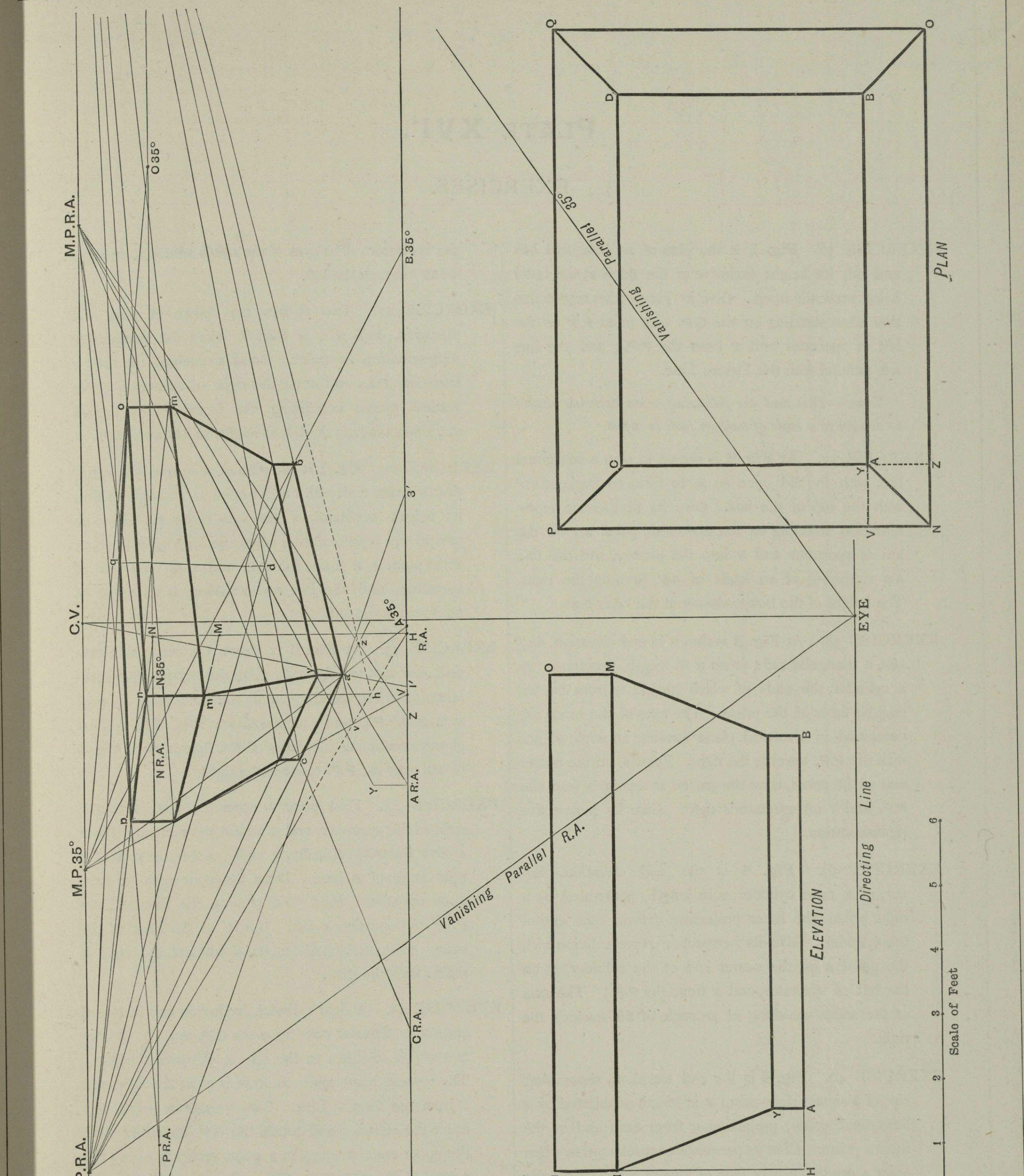

M.P.R.A.
C.V.
M.P.35°
V.P.R.A.
O35°
B.35°
3'
A35°
H
R.A.
I'
Z
A'R.A.
C R.A.
P R.A.
N35°
N R.A.
Vanishing Parallel R.A.
Vanishing Parallel 35°
EYE
Directing Line
PLAN
ELEVATION
Scale of Feet
0
1
2
3
4
5
6

PLATE XVI*.

EXERCISES.

EXERCISE 36. **Fig. 1** is the plan of an octagonal box and lid, its height exclusive of the lid is 2′, the latter being vertically open. Give its perspective representation when standing on the G.P. with point A, 2′ on the left of spectator and 3′ from the P.P.; and the line A B, parallel with the Picture Line.

Note.—*This and the following seven exercises should be drawn to a scale of half an inch to a foot.*

EXERCISE 37. At **Fig. 2** is shown in plan a hexagonal box with the lid open so as to form an angle of 90° with the top of the box. Give its perspective representation standing on the G.P. with point A, 1′ on the left of spectator and 5′ into the picture, and the line A B vanishing at an angle of 45° towards the right. The height of the box exclusive of the lid is 2′ 6″.

EXERCISE 38. At **Fig. 3** is shown in end elevation, *half size*, a triangular right prism 3′ in length, penetrated by a cylinder, the ends of which project 1½′ from the triangular faces of the prism. The axes of the solids are contained in a vertical plane forming an angle of 50° with the P.P. towards the right. Point A, on the nearer end of the prism, is on the ground at a point 2′ from the P.P. and 1′ on spectator's right. Give its perspective representation.

EXERCISE 39. **Fig. 4** is the end elevation, *half size*, of a right cylinder 3′ in length, penetrated by a right prism, the latter projecting 1½′ from each end of the cylinder. Give its perspective representation with the point A on the nearer end of the cylinder, 3′ on the left of spectator, and 6′ from the P.P.. The axes of the solids vanishing at an angle of 65° towards the right.

EXERCISE 40. **Fig. 5** is the end elevation, shown *half size*, of a square right prism 3′ in length penetrated by a hexagonal prism, projecting 1½′ from each end of the square prism. Give its perspective representation when point A on the nearer face of square prism is on the ground plane 2′ beyond the P.P., and 2′ on the right of the spectator. The axes of the solids vanish at an angle of 55° towards the left.

EXERCISE 41. Two figures are shown in plan and elevation, *half size*, at **Fig. 6**. Give the perspective representation as under. Point A touching the G.P. 2′ from the P.L. and 4′ on the right of spectator. The vertical planes containing the faces of the figures vanishing towards the left at angles of 45° with P.P..

EXERCISE 42. **Fig. 7** is the plan of a square box, cylindrical inside and with the lid open at an angle of 90°; its height (exclusive of the lid) being 3′. Give its perspective representation when standing upon the G.P. with point A, 3′ from the P.L. and 2′ on the left of spectator; and the line B A vanishing at an angle of 65° towards the left.

EXERCISE 43. At **Fig. 8** is shown by plan and elevation, *half size*, two figures. Give the perspective representation when standing upon the G.P., point A being 5′ from the P.L. and 1′ on spectator's left. The vertical planes containing the faces of the figures form angles of 40° with the P.P. towards the right.

EXERCISE 44. Find point A upon the G.P., 3′ to the right of the spectator, and 6′ within the picture. From A draw line A B, vertically, 9′ high. A B is the axis of a right cone of 8′ base. Draw the cone with its base upon the G.P.. Find point C upon the Picture Line, 6′ on the spectator's left. Join C to the apex of the cone. Give the actual length of C B and the angle it makes with the G.P..

EXERCISE 45. A right cylinder, having an axis of 9′ and ends 5′ in diameter rests upon the G.P. in a line of contact which vanishes to the left at 40° with the P.P.. The nearest point upon the G.P. is 3′ to the right and 6′ from the Picture Line. Two rods 15′ in length rest upon the cylinder and touch the G.P. on either side, the upper ends meeting in a point vertically above the centre of the cylinder. The rods are parallel to the ends of the cylinder.

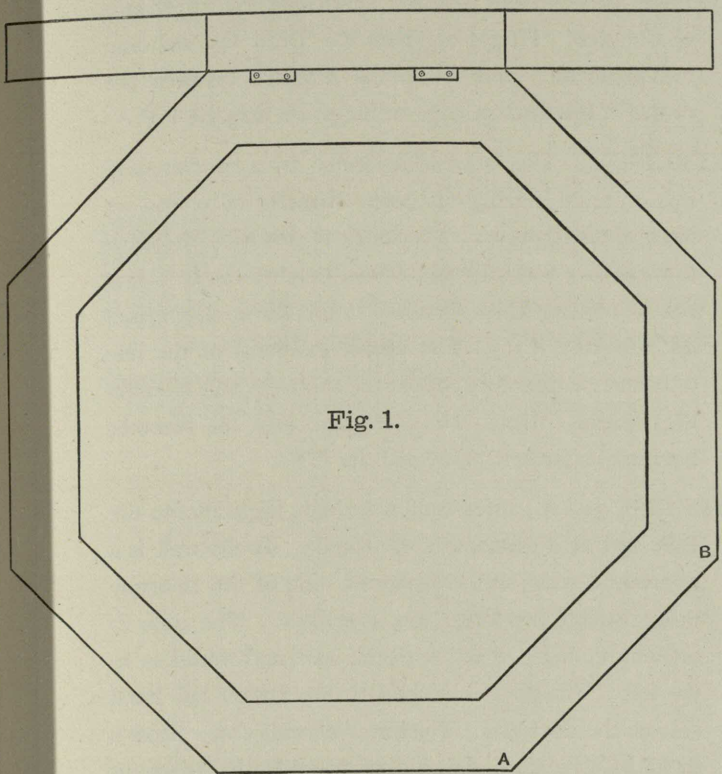


Fig. 1.

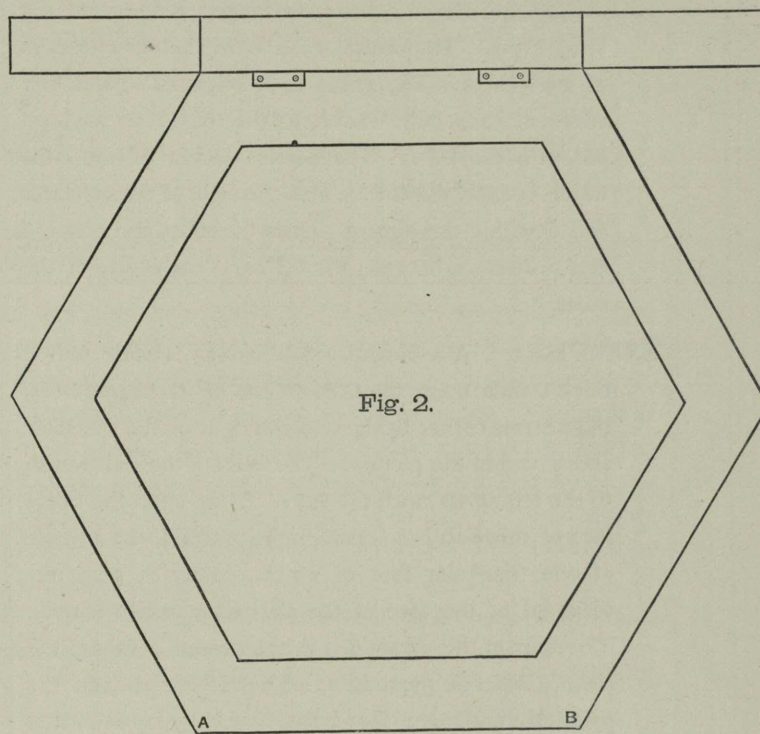


Fig. 2.

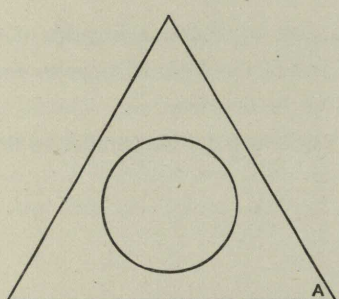


Fig. 3.

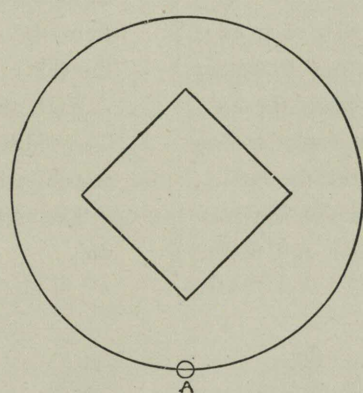


Fig. 4.

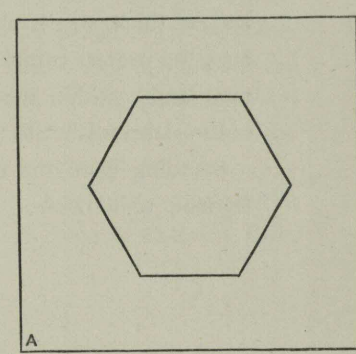


Fig. 5.

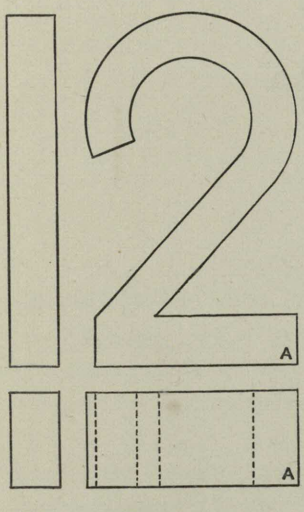


Fig. 6.

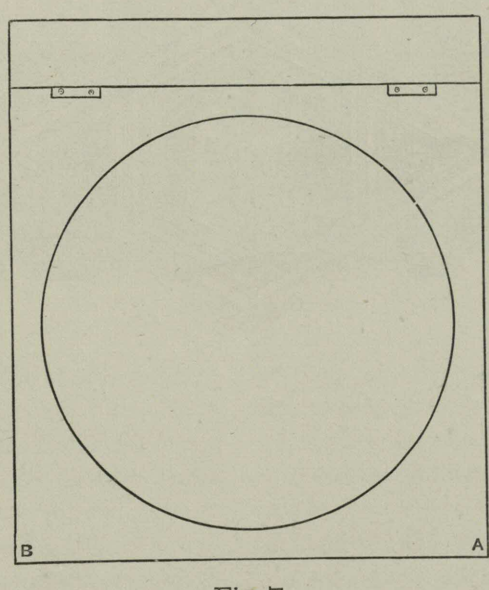


Fig. 7.

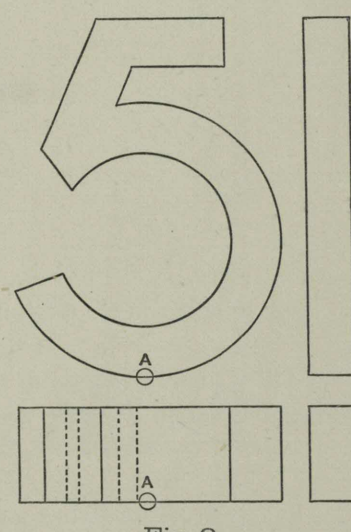


Fig. 8.

EXERCISE 46. Seven cubes are arranged in the form of a solid cross. The central cube is in contact with each of the others. The cubes have edges of 3′, and four edges of each cube vanish to the left at an angle of 35° with the P.P.. The nearest corner of the cube which is upon the G.P. is 3′ to the right of the spectator and 6′ within the picture. Draw the cubes and give the actual distance between the P.P. and the centre of the cross.

EXERCISE 47. A rectangular slab, 6′ long, 4′ wide, and 2′ thick stands upon the G.P. on one of its largest faces, the nearest corner being 4′ to the right of the spectator and 3′ within the picture. The ends of the slab vanish to the left at 40° with the P.P.. Lying upon the upper face of this slab is a square, right pyramid, the altitude of one triangular face of which, coincides with one diagonal of the face of the slab upon which it rests. The apex of the pyramid is in the nearest corner of the slab. Base of pyramid 6′. Draw the slab and the pyramid, and give the actual distance between the centre of the pyramid and the centre of the slab.

EXERCISE 48. An equilateral triangular slab of 9′ edge and 2½′ thick, stands upon the G.P. on one triangular face, the nearest corner being 4′ to the left of the spectator and 4′ within the picture. One rectangular face of the slab vanishes to the right at 40° with the P.P..

Standing upon the upper face of this slab is a pentagonal, right prism. A diagonal of one end of the prism is coincident with the altitude of the upper face of the slab. Height of prism 6′. Draw the slab and the prism and give the actual distance between the centre of the furthest edge of the prism and the P.P..

EXERCISE 49. The letter O is formed by a circular solid ring 3′ thick, having an outer diameter of 11′ and an inner diameter of 6′. The letter stands with its plane surfaces in a vertical position and touches the G.P. in a line of contact which vanishes to the left at an angle of 45° with the P.P.. The nearer extremity of the line of contact is directly opposite the spectator and 9′ within the picture. Draw the letter and give the distance between its farthest point and the P.P..

EXERCISE 50. A garden wall is 7½′ high, is parallel to the P.P. and at a distance of 7½′ from it. In the wall is a doorway 5′ wide and 6′ high, one side of the doorway being directly opposite the spectator. The door is open at an angle of 65° with the wall, and vanishes to the left. The door is hinged to the further left hand side of the doorway. Thickness of wall 1′ 6″. Show a flight of five steps descending towards the spectator from the doorway to a path at a level of 2½′ below the G.P..

Each step to have a riser of 6″ and a breadth of 1′. The width of the path and of the flight of steps to correspond with the width of the doorway.

The vertical face of each step to be parallel to the wall.

THEORY.

HAVING now some knowledge of the methods employed in solving Perspective Problems, we may further investigate the question as to how far our drawings are in accordance with the natural appearances of the objects represented.

It will be remembered that at the commencement of our study, it was stated that the science of Perspective is founded upon two other sciences, viz., Geometry and Optics. To understand more of the theory of Perspective, it will be necessary to acquire some further knowledge of the sciences upon which Perspective is based.

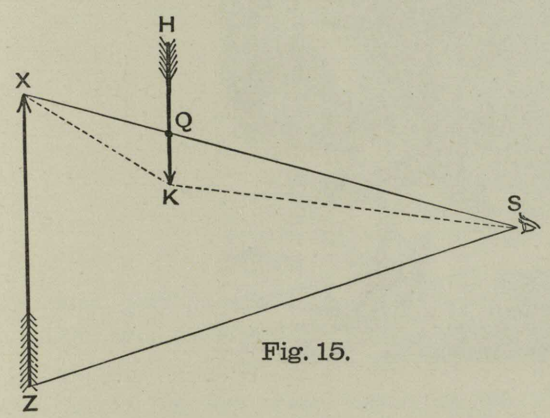


Fig. 15.

And first of Optics. It was asserted (see page 7). 1. That we see by the agency of light which passes from objects to our eyes in straight lines which are called Visual Rays.

We may fairly realise that we see by the agency of light, as all objects, except such as may be styled self-luminous, when placed in a dark chamber, are not perceivable by us, except by touch, smell, or hearing; we cannot *see* them, they are invisible. But, when by removing a shutter or igniting a flame, we introduce something to the chamber which was not present when the chamber was dark; we become at once conscious of the appearance of the object; we perceive it by the sense of sight.

This something which must always be present to enable us to see, is called Light.

That the Visual Rays pass from objects in straight lines to the eye may be proved by the following experiments:—

In Fig. 15, X Z is an arrow, seen by the eye at S. If we stretch a thread in a straight line from Z to the eye, it will represent a Visual Ray. Then if another object, as H K, be interposed, as soon as it touches the thread at Q, the points of the two arrows will appear to be in contact. Then if the arrow H K be still further interposed, *covering point* X, X will not be seen and the thread will be no longer one straight line but two, X K and K S.

The fact may also be proved as follows:—Pierce two screens with a large pin, and place them so that the holes

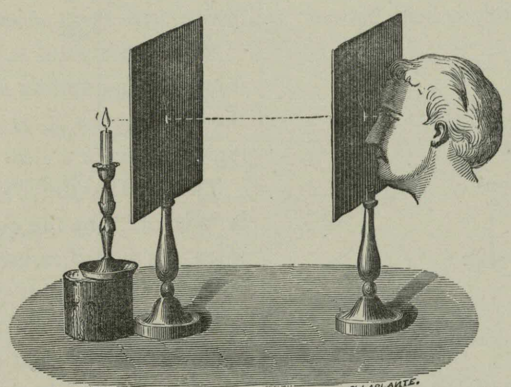


Fig. 16.

are in a straight line with a flame, as the light of a candle or lamp. On fixing the eye to one of these holes we are able to see the flame; but if we slightly move the flame, one of the screens, or the eye, the flame is no longer visible. To be visible, the flame, the holes in the screens, and the eye must all be in the same straight line (see Fig. 16).

It should however be noted that when light passes through a medium of a different density than air the direction of the rays is changed.

Knowing then that we see by the agency of light which in an ordinary medium such as air, travels in straight lines, we may inquire how light transmits the impressions of external objects to the observer's eye.

If we exclude all the light from a room except what may

enter through a small hole pierced in one shutter, and place a white screen opposite the hole, a picture of the external objects will be formed upon the screen (see Fig. 17). This picture will be an exact facsimile of the external objects, but they will appear to be *upside down*. This may be readily explained (see Fig. 18). M N is the screen, O the hole in the shutter, A B an object, and A′ B′ the picture of that object upon the screen. From the point of the arrow A Visual rays proceed in all directions; one of which, after passing through the hole in the shutter, comes in contact with the screen

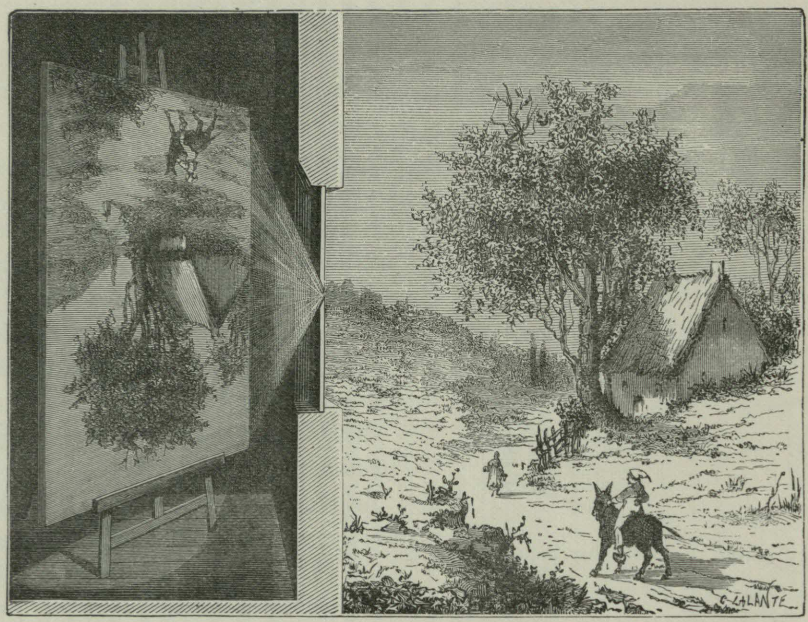


Fig. 17.

at A′. A′ can only receive light from A, and A gives light only to that point on the screen at A′, therefore A′ is the picture or image of A. In the same way B′ is the image of B, and if we imagine the rays passing from every part of the object through the hole in the shutter, they will give the whole of the image between A′ and B′. Thus, an *inverted* representation of the arrow is formed upon the screen.

If a card pierced with a large pin hole is held between a

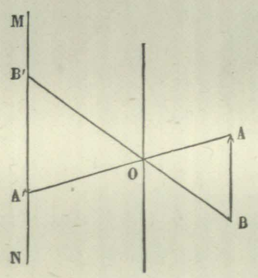


Fig. 18.

flame and a screen an inverted image of the flame will be seen upon the screen.

The human eye may be roughly described as a chamber of a spherical or globular form, with a circular opening in front. This circular opening is called the *pupil*, and through it the visual rays pass to the interior of the eye. The visual rays passing from space in all directions through the small pupil, are received upon what may be called the *interior wall* of the globular chamber forming the eye (see Fig. 19). This interior wall is called the *retina*, and upon it the impressions of external objects are received, just as they are received upon the screen in a dark chamber. These impressions are conveyed by the optic nerve, from the retina, to the brain.

In front of the *pupil* is a segment of a small sphere composed of the *cornea* and the *aqueous humour*, both of which are transparent, and from their shape and density have a convergent effect upon the rays passing through them.

Behind the pupil is the transparent *crystalline lens*, which from its shape and its elasticity, is a powerful agent in aiding the convergence of the rays, and in bringing objects at varying distances to a clear focus upon the retina.

The pupil has the power of contraction and dilatation, which is influenced by the quantity of light entering the eye, but when it is dilated to the utmost, its size is very small in comparison with the great chamber forming the body of the eye.

In Fig. 19 we have a rough sectional diagram of the eye and an object in front of it. This object, an arrow, is seen by means of the Visual rays proceeding from it, the principal two of which are shown. The Visual ray from A passes through the pupil and is received upon the retina at a. In the same way the Visual ray from B passes through the pupil and is received upon the retina at b. It will thus be seen that the impressions or images received upon the retina are *inverted;* but by long reasoning and experience, the mind has acquired the habit of determining the real positions of objects, and does not, though the image is so received, imagine them to be upside down.

It will also be observed, in the same way, that that portion of an object which is upon the *right* will be pictured upon the retina on the *left*, and vice versa, but the mind, for the reasons before stated never imagines the object to be reversed. This fact is another proof that, as mentioned at the commencement of our study, to see accurately is a matter of education and practice.

We have seen in Book I., page 8, that by determining the points in which the Visual Rays passing from any object, intersect the Picture Plane, we gain a Perspective delineation of that object. These Visual Rays pass from the object to a single point which represents the observer's eye.

But the observer, as a rule, has *two* eyes, with each of which he sees anything coming within range of his vision.

We will return to our coin as an illustration of this fact. Let us hold the coin between the thumb and the first finger

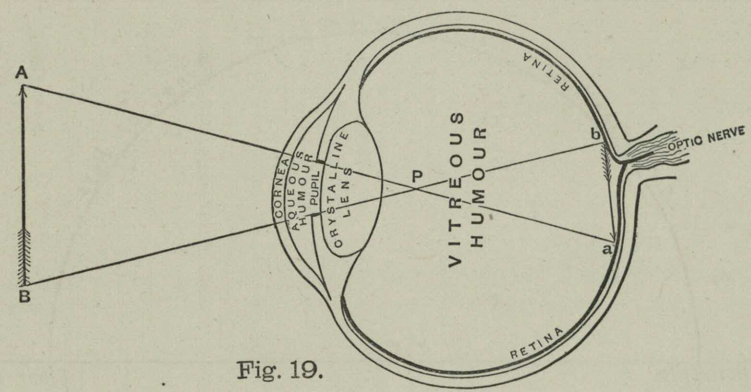


Fig. 19.

in an upright position, about 18 inches in front of the *centre* of the face, the edge being presented towards the face. *On closing the left eye,* we see the edge and a very small portion of the right hand circular surface. Then still holding it in the same position, but closing the right *eye* and opening the left, we see the edge and a small portion of the *left hand* circular surface, the *right hand* circular surface having passed from view. To illustrate the same point, let us hold up one finger between our eyes and a wall. On looking at the wall we see two fingers, one being seen by the right eye and the other by the left. But now, on taking our eyes from the wall and looking at the finger we see only one. Yet on adopting the method employed in the case of the coin we find we see more

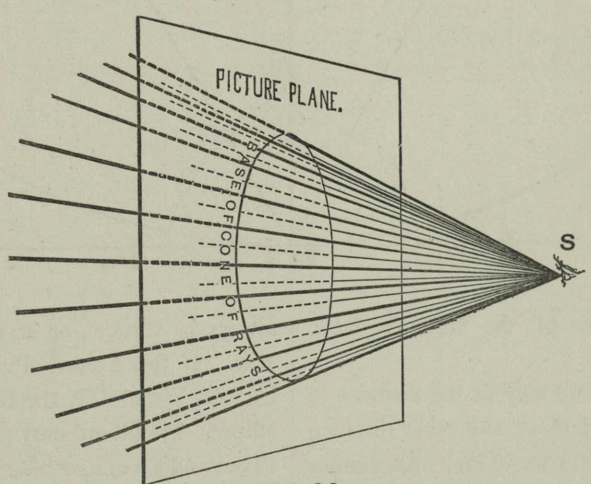


Fig. 20.

of the finger nail with one eye than we see with the other.

We thus see that with each eye we have a different view of the same object.

But our Perspective drawings do not give two different views of the same object. How then can they be truthful representations of nature?

The reason is that although a spectator using both eyes sees two slightly different views of the same object, the mind forms from the two views, an impression of *one* distinct object, or *one* mental image.

Perspective, then, regards one point as representing the eye of the spectator, and the Visual rays are represented by straight lines.

On reflection it will be seen that the external Visual rays passing through the circular pupil, form a cone, or cone of rays. Anything beyond reach of the external rays, or lying outside this cone of rays, would be invisible to the spectator.

The angle of this cone of rays, or the *angle of vision,* varies, of course, in different eyes, but the average angle is 60′. The Central Visual Ray forms the axis of the cone of rays, and the Picture Plane situated at right angles to the Central Visual Ray, would give a circular base to the cone (see Fig. 20).

On referring to Fig. 21 we have a representation of the Horizontal Line, the Ground Line, the Central Visual Ray, the Eye, the Directing Line, and the Circle forming the Base of the Cone of rays drawn to a scale of 4 feet to an inch. Anything represented outside this Circumscribing Circle or Base of the cone of rays, would appear distorted and unnatural. In order to be able to readily determine the size of this Circumscribing Circle it may be taken as a rule that the distance between the Eye and the centre of vision should be equal to the *altitude* of an equilateral triangle, *one side*

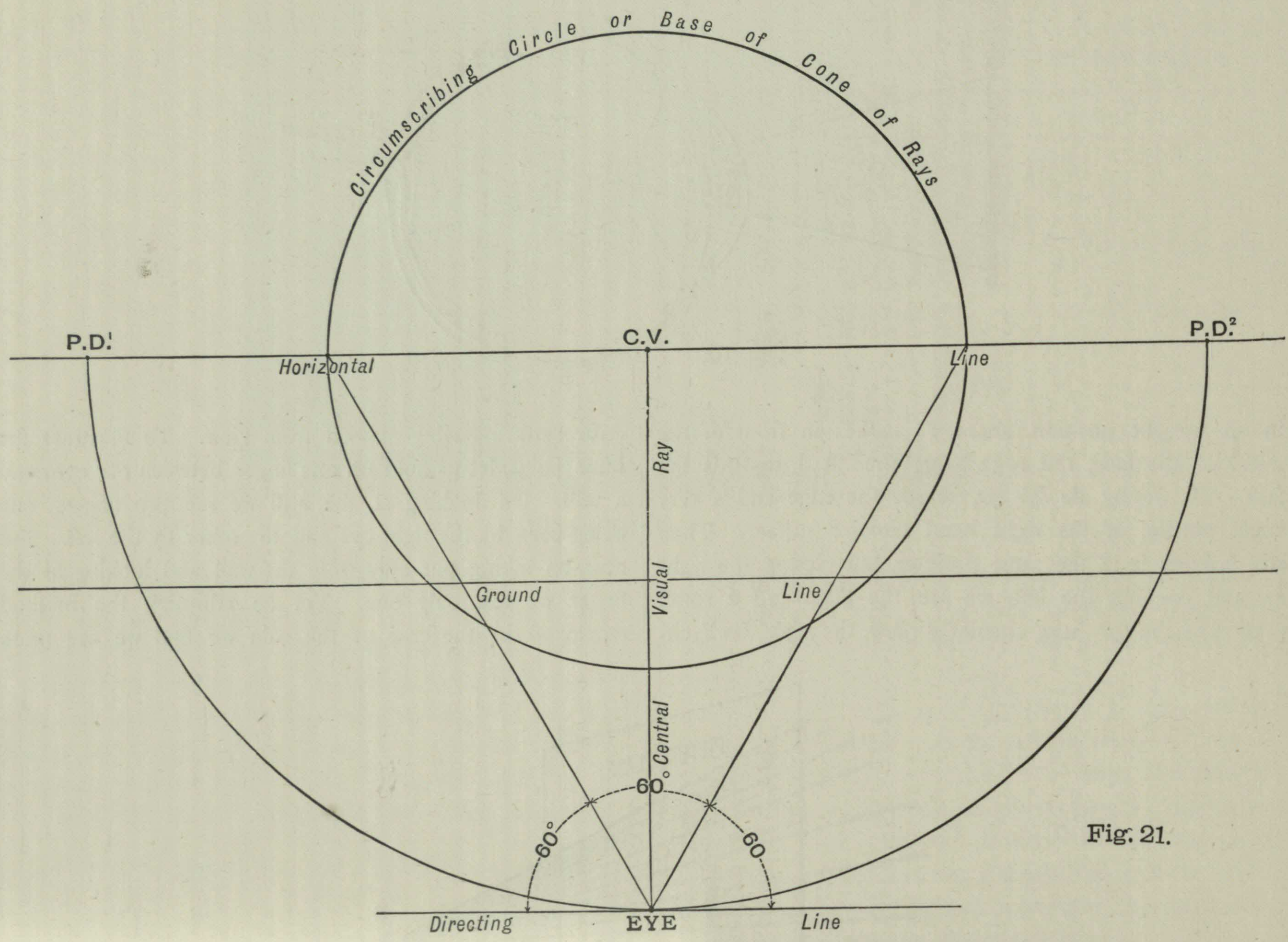


Fig. 21.

of which would be the diameter of the Circumscribing Circle.

If then, we remember this rule we may fix the distance of the eye as either 12 inches, 12 feet, or any other distance from the Picture Plane, only taking care to limit the amount drawn, by the Circumscribing Circle, in order to avoid distortion.

It is not an absolute law that the Picture Plane should always be vertical, or at right angles to the Central Visual Ray. If the Picture Plane is not at right angles to the Central Visual Ray the base of the cone of rays will be an ellipse. For all ordinary purposes however, a vertical Picture Plane and a circular base of cone of rays are all required.

The theoretical portion of our study will be further continued in Part III.

POYNTER'S SOUTH KENSINGTON DRAWING-BOOK.

ELEMENTARY PERSPECTIVE DRAWING.

BY

S. J. CARTLIDGE, F.R.HIST.S.

PART III.

PLATE XVII.

IN the preceding division of our study, lines inclined to the G.P. were drawn by making use of *vertical* lines. (See Problems 9, 11, 15, 16, 17, 20, 21, 22 and 23.) We must now consider the method of drawing lines inclined to the G.P., by means of their Vanishing and Measuring Points. These points will usually fall wide of the H.L., and hence are called "*accidental*" vanishing and measuring points.

If we have a line inclined at a certain angle to the G.P., and wish to find its V.P., we must imagine a *vertical plane* to pass *through* the line. For example, in Fig. 22 the block ABCDEF, has its square end A, F, E, C, in a vertical position; and its oblong face ACDB, also in a vertical position. If we draw the diagonal CF, of the square end, it is *contained* in the *vertical plane* of the square end, which is receding from the spectator, to the left. It is, of course, quite obvious that the diagonal CF, is *inclined to the* G.P.. In the same way the diagonal AD, of the *oblong* face is contained in the *vertical plane* of that face, which is vanishing to the right. If we produce CE, and AF, to the H.L., we shall have the vanishing point of two *horizontal lines* which are contained in the *vertical plane* of the *square* end of the block. This V.P. is called the *Original Vanishing Point* of the vertical plane of the end of the block. Through this V.P. a *Vertical Vanishing Line* is drawn, upon which the Vanishing Points of any lines contained in the end of the block, or in a vertical plane passing through the end of the block, will fall. If we produce CF, to meet the Vertical Vanishing Line, its V.P. will be found *below* the H.L. in A.V.P. of CF.

This V.P. is below the H.L. because the line is vanishing in a *downward* direction, *it descends*. The V.P. of AD, would be determined in the same way, viz.:—by producing CD and AB to meet the H.L. in the Original Vanishing Point, drawing the Vertical Vanishing Line through the Original Vanishing Point, and producing AD to meet the Vertical Vanishing Line.

The V.P. of AD would be *above* the H.L. because AD *ascends* from the spectator. If an inclined line has its nearer extremity lower than its farther one, the line *ascends*. If, on the contrary, the *nearer* extremity is higher than the farther one, the line *descends*.

PROBLEM 24. *A right line* 9′ *long, has one extremity upon the* G.P. *at a point* 2′ *to the left of the spectator and* 4′ *within the picture. The line is contained in a vertical plane which recedes from the* P.P. *at an angle of* 50° *to the right. The line is inclined at an angle of* 45° *to the* G.P. *and vanishes upwards from the spectator.*

HAVING found the nearer extremity x, of the line. By drawing an *Original Vanishing Parallel* at 50° with the P.P. to the right, we find the Original Vanishing Point of the vertical plane containing the line, in Original V.P. 50°. Through Original V.P. 50° draw the V.V.L. (*Vertical Vanishing Line*).

To find the Accidental Vanishing Point of the line. Find Original M.P. 50° by Proposition VII., Page 14.

Proposition IX. *To find the Vanishing Point of an inclined line.* FROM THE ORIGINAL M.P. DRAW A LINE TO MEET THE V.L., PARALLEL TO THE LINE WHOSE V.P. IS REQUIRED.

Then, as the line is inclined at an angle of 45° to the G.P., which is horizontal, draw from Original M.P. 50° to meet the V.V.L., an *Accidental Vanishing Parallel at an angle of 45° with the* H.L.. This gives A.V.P. 45°.

Join x to A.V.P. 45°.

Now that we have the line in its proper direction, it only remains for us to measure it.

Proposition X. *To find the Measuring Point of an inclined line.* UPON THE V.L. SET OFF FROM THE A.V.P. THE LENGTH OF THE ACCIDENTAL VANISHING PARALLEL.

Take then, *Accidental Vanishing Parallel 45°* as radius, and A.V.P. 45° as centre, giving A.M.P. 45° on the V.V.L.. We must now transfer X (Prop. V.) to the P.P. from its M.P.. It will of course be transferred to the P.P. in the Intersecting Line *of the Vertical Plane which contains the line to be measured.*

On reference to **Fig. 22** it will be seen that the intersection of the square end of the block with the P.P. would be found by producing edge FA, (the Ground Intersection of the square end) to meet the G.L.. Through the point so found upon the G.L. the intersection of the square end with the P.P. would be drawn *vertically*, as vertical planes meet each other vertically.

Draw then a Ground Intersection from Original V.P. 50° through x to meet the ground line in I.

Through I, the point in which the Ground Intersection meets the G.L., draw a *Vertical Intersecting Line.*

Transfer x from A.M.P. 45° to its I.L. in X 45°.

From X 45° measure the length, 9′, in 9.45°.

Return 9.45°, to A.M.P. 45° and it cuts off 9′ in z.

EXERCISE 51. A right line 7′ long has one extremity upon the G.P. at a point 3′ to the right of the spectator and 3′ within the picture. The line is inclined upwards at an angle of 30° to the G.P. and is contained in a vertical plane which vanishes to the left at an angle of 30° with the P.P..

Fig. 22.

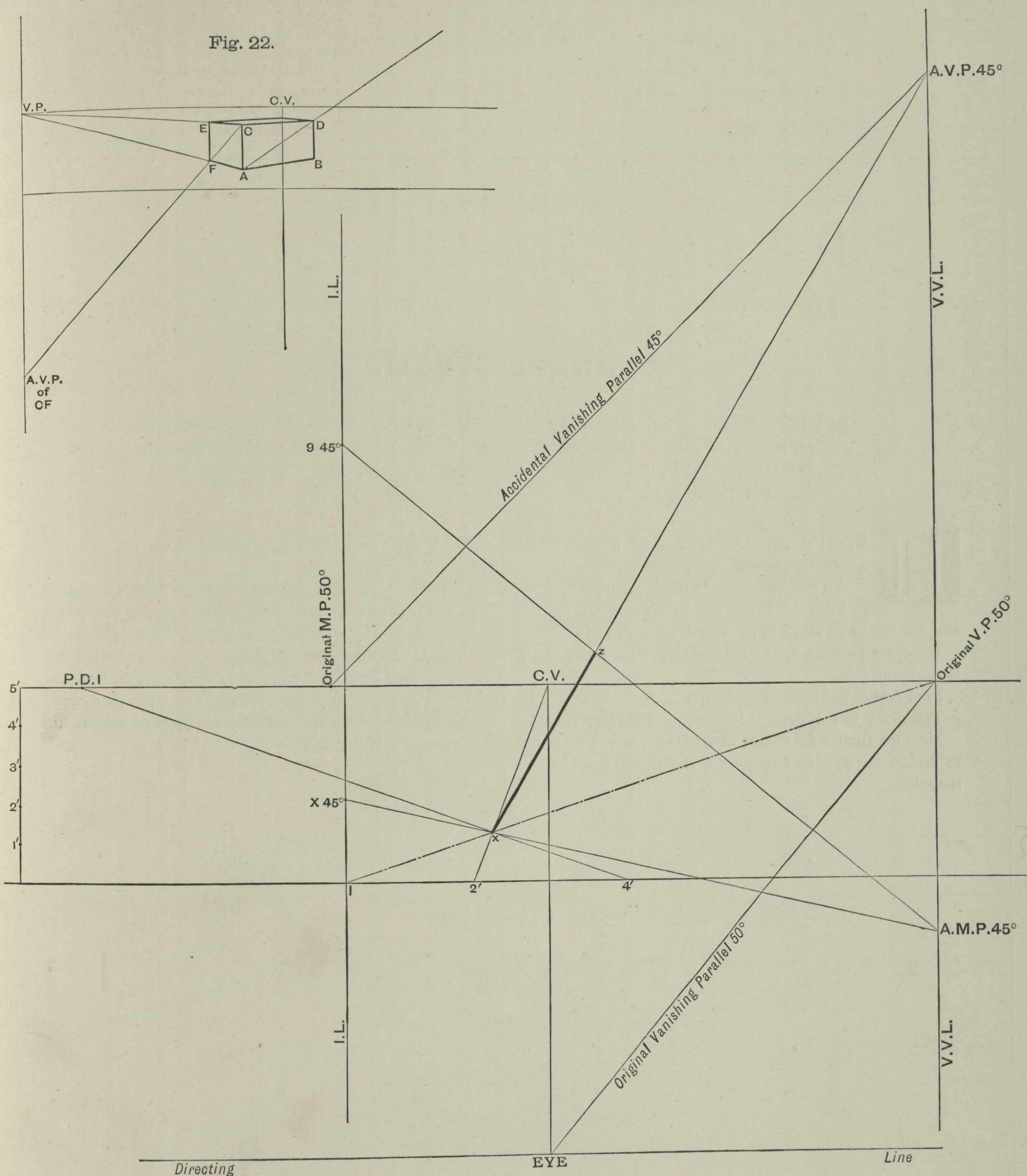

V.P.
C.V.
E
C
D
F
A
B
A.V.P.
of
CF
A.V.P.45°
V.V.L.
I.L.
Accidental Vanishing Parallel 45°
9 45°
Original M.P.50°
Original V.P.50°
z
P.D.I
C.V.
5′
4′
3′
2′
1′
X 45°
x
I
2′
4′
A.M.P.45°
Original Vanishing Parallel 50°
I.L.
V.V.L.
Directing
EYE
Line

PLATE XVIII.

PROBLEM 25. *An equilateral triangle of* 7′ *side has one corner upon the* G.P. *at a point* 3′ *to the right of the spectator and* 5′ *within the picture. One side is inclined at an angle of* 40° *to the* G.P.. *The triangle is contained in a vertical plane which recedes from the* P.P. *at an angle of* 40° *to the left.*

HAVING drawn one side x y of the triangle by the process explained in Problem 24.

To find the second side of the triangle.

Find A.V.P. 2nd Side by drawing *Acc. Van. Parallel 2nd Side* at an angle of *60°* with *Acc. Van. Parallel 40°.*

Draw a line from A.V.P. 2nd Side through y, producing it towards the P.P..

Find A.M.P. 2nd Side. Proposition X.

Transfer Y to the Vertical Intersecting Line in Y 2nd Side.

Measure from Y 2nd Side, 7′, to 7′ 2nd Side, return 7′ 2nd Side to A.M.P. 2nd Side, and it gives z, the third corner of the triangle.

The triangle is completed by joining z to x.

EXERCISE 52. Draw the same triangle with its nearest corner upon the G.P. at a point 3′ to the left of the spectator and 5′ within the picture. One side is inclined at an angle of 60° to the G.P., the triangle being contained in a vertical plane which recedes from the P.P. at an angle of 30° to the right. Give the actual distance between the centre of the furthest side of the triangle and the P.P..

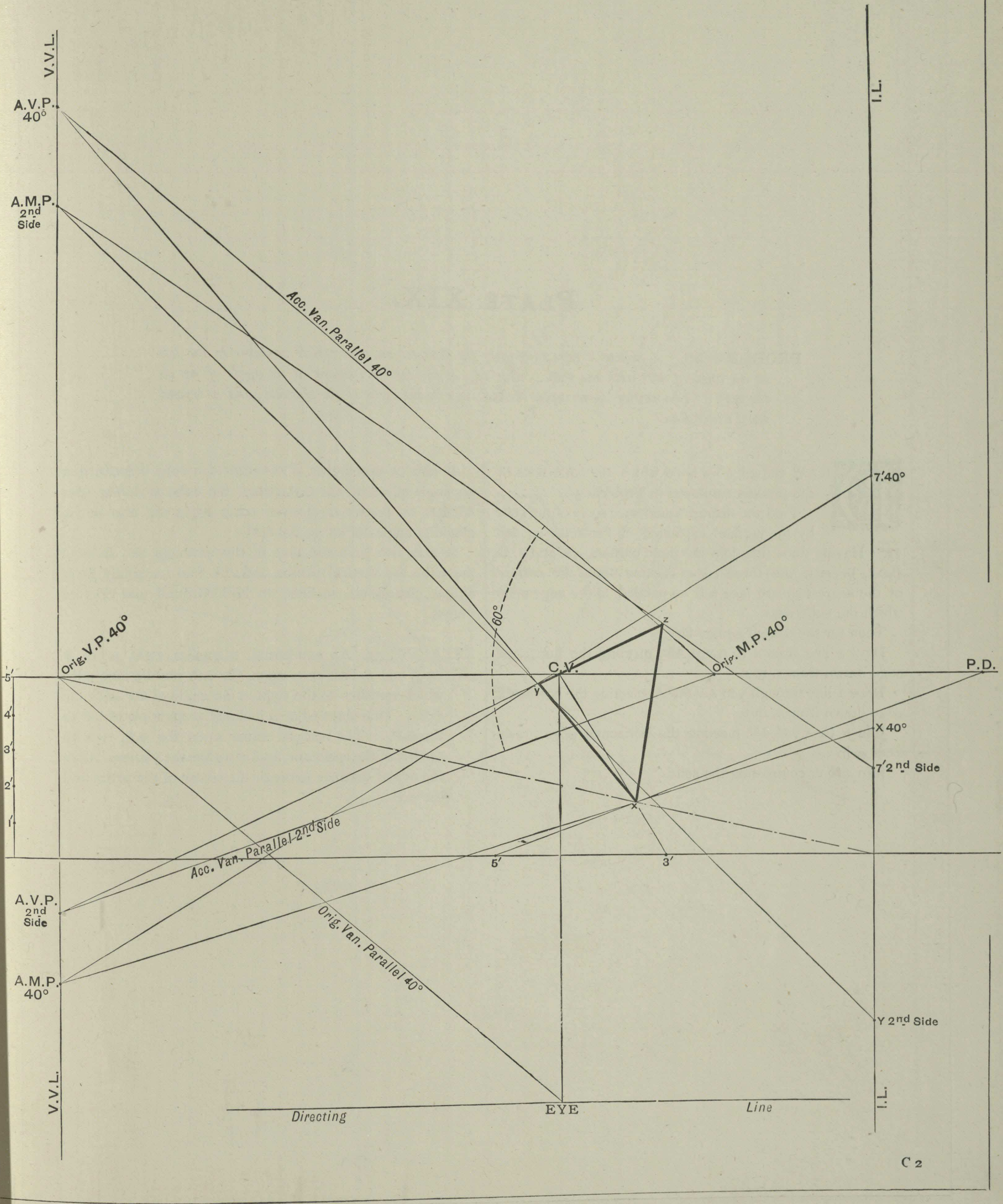

V.V.L.
A.V.P. 40°
A.M.P. 2nd Side
I.L.
Acc. Van. Parallel 40°
7' 40°
60°
z
Orig. V.P. 40°
C.V.
Orig. M.P. 40°
P.D.
y
5'
4'
3'
2'
1'
X 40°
7' 2nd Side
x
Acc. Van. Parallel 2nd Side
5'
3'
A.V.P. 2nd Side
Orig. Van. Parallel 40°
A.M.P. 40°
Y 2nd Side
V.V.L.
I.L.
Directing
EYE
Line

PLATE XIX.

PROBLEM 26. *A cube of* 6′ *edge rests upon the* G.P. *on one edge which vanishes to the left at an angle of* 45° *with the* P.P.. *One edge of the cube is inclined at an angle of* 45° *to the* G.P.. *The nearest corner upon the* G.P. *is* 2′ *to the right of the spectator, and* 6′ *beyond the Picture Line.*

DRAW the edge x a upon which the cube rests by the process employed in Problem 3.

Find the nearest square face w x y z of the cube by the method explained in Problems 24 and 25. Having done this, join the three corners, z, y, and w, to P.D.1, because the three edges starting from the corners of the nearest square face will be parallel to the edge upon which the cube rests.

Then for the distant square face.

Draw a line from a to A.V.P. 45°, meeting the horizontal edge drawn from z, in d.

Draw a line through a to A.V.P.R.A. meeting the horizontal edge drawn from y, in b.

Join b to A.V.P. 45° meeting the horizontal edge drawn from w, in c.

Join c to d, completing the solid.

It may be noted that, if *one* edge of a cube is inclined at an angle of 45° to the G.P. (when the cube is *resting upon the* G.P. *on another edge*) seven other edges will also be inclined at angles of 45° to the G.P.

It may also be noted that in this problem two faces of the cube are vertical planes, and the four remaining faces *oblique* planes, viz. inclined to both Ground and Picture Planes.

EXERCISE 53. An equilateral, triangular, right prism, 8′ long, and 5′ base rests upon the G.P. on one long edge which vanishes to the right at an angle of 30° with the P.P.. One short edge is inclined at an angle of 30° to the G.P.. The nearest corner upon the G.P. is 3′ to the left of the spectator, and 7′ within the picture. Give the actual distance between the centre of the prism and the G.P..

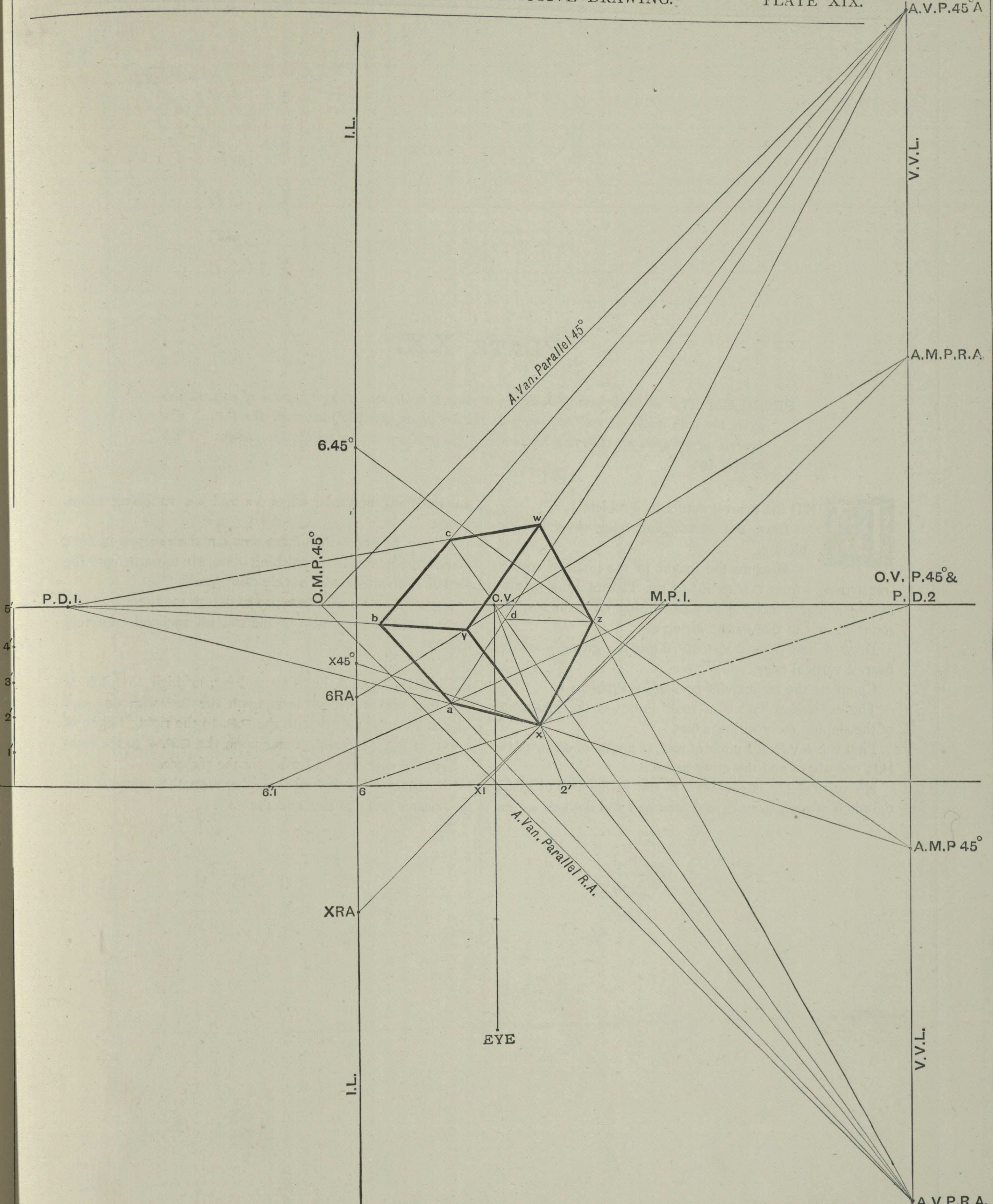

A.V.P.45° A
V.V.L.
I.L.
A.Van. Parallel 45°
A.M.P.R.A.
6.45°
w
c
O.M.P.45°
O.V. P.45° &
P. D.2
P.D.I.
C.V.
M.P.I.
b
d
z
y
X45°
6RA
a
x
5
4
3
2
1
6.I
6
XI
2′
A.Van. Parallel R.A.
A.M.P 45°
XRA
EYE
V.V.L.
I.L.
A.V.P.R.A.

PLATE XX.

PROBLEM 27. *A block form of building,* 15′ *long,* 9′ *wide, and* 9′ *high to pitch of roof, stands upon the* G.P. *with one end vanishing to the left at an angle of* 50° *with the* P.P.. *The nearest corner upon the* G.P. *is* 3′ *to the left of the spectator, and* 3′ *within the picture. Pitch of roof* 45°.

FIND the nearest corner by Problem 1.

Draw the rectangular base abyx, by Problem 5.

Measure the height to the pitch of roof by transferring y from O.M.P. 50° to the Picture Line in 9′.50, raising a line of heights at 9′.50, setting off 9′ in 9′V, and returning 9′V to O.M.P. 50°, giving z,

Draw a line from O.V.P. 50° through z giving w on the nearest vertical edge.

Complete the rectangular portion by Problem 10.

The expression "pitch of roof" indicates the inclination of the sloping faces with the G.P..

Find the A.V.Ps for pitch of roof, at angles of 45° with the H.L. one above and the other below it.

No vertical I.L. is required, as the inclined lines giving the pitch of roof, *measure each other in* v and e, edges w v and d e, vanishing upwards, edges v z and e c, vanishing downwards.

It may be noted that faces w v d e, and v z e c, are oblique planes, the nearer face vanishing obliquely upwards, and the further one vanishing obliquely downwards.

It would be a useful exercise to determine the actual distance between the centre of the nearest inclined edge and the P.P..

EXERCISE 54. A building 14′ long, 10′ high to pitch of roof, and 6′ wide, stands upon the G.P. with one end at an angle of 30° with the P.P. to the right. Pitch of roof 35°. Nearest corner upon the G.P. 4′ to the right of the spectator and 9′ within the picture.

Give the actual distance between the centre of the nearer inclined face and the P.P..

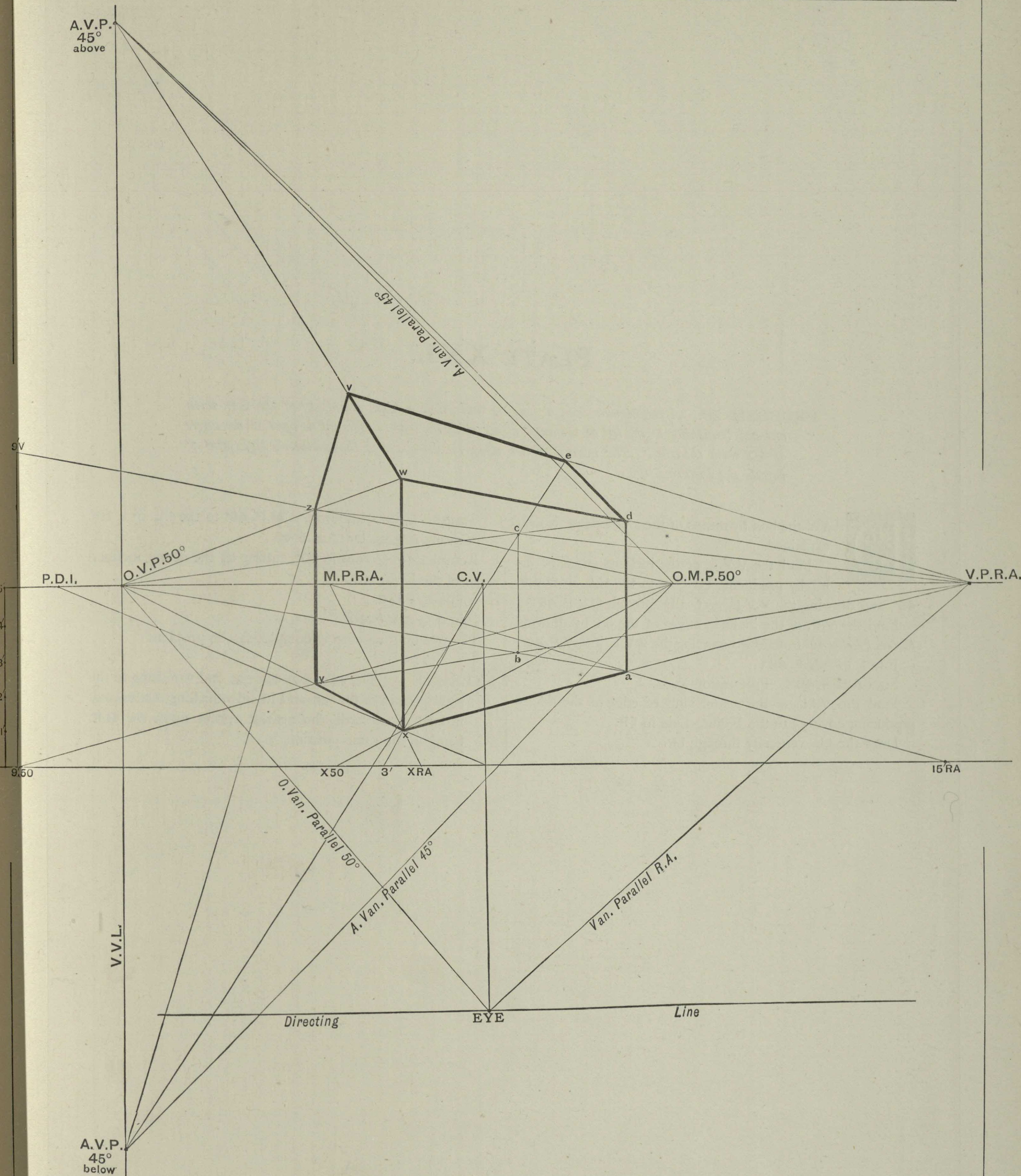

A.V.P.
45°
above
A. Van. Parallel 45°
v
e
w
d
z
c
O.V.P.50°
P.D.I.
M.P.R.A.
C.V.
O.M.P.50°
V.P.R.A.
b
a
y
x
X50
3'
XRA
15'RA
O. Van. Parallel 50°
A. Van. Parallel 45°
Van. Parallel R.A.
V.V.L.
Directing
EYE
Line
A.V.P.
45°
below

PLATE XXI.

PROBLEM 28. *A rectangular box,* 9′ *long,* 5′ *wide, and* 3′ *high, stands upon the* G.P. *with one end vanishing to the left at an angle of* 45° *with the* P.P.. *The lid is open at an angle of* 40° *with the* G.P.. *The nearest corner upon the* G.P. *is* 2′ *to the spectator's right and* 3′ *beyond the Picture Line.*

DRAW the box by means of the methods previously explained.

Find the V.V.L. of the inclined edges of the lid by producing edge x y to the H.L. in O.V.P. 45°, and drawing the V.V.L. vertically through O.V.P. 45°.

Find the V.P. of the inclined edges of the lid by drawing *A. Van. Parallel 40°* from M.P.I meeting the V.V.L. in A.V.P. 40°.

Join L to A.V.P. 40°.

Find A.M.P. 40°. Proposition X.

Find the V.I.L. of the nearer inclined edge of the lid by producing edge x y to the Picture Line in I.P..

Draw the I.L. vertically through I.P..

Transfer L, by a line from A.M.P. 40° to the I.L. in L 40°.

Measure 5′ from L 40° in 5′40°.

Return 5′40° to A.M.P. 40°, cutting off the nearer inclined edge of the lid in b.

Join b to P.D.2.

Join D to A.V.P. 40°.

The two lines meet in d, completing the problem.

EXERCISE 55. The same box has one end vanishing to the right instead of the left and its lid vanishing *downwards* instead of upwards, the nearest corner upon the G.P. being in the same position.

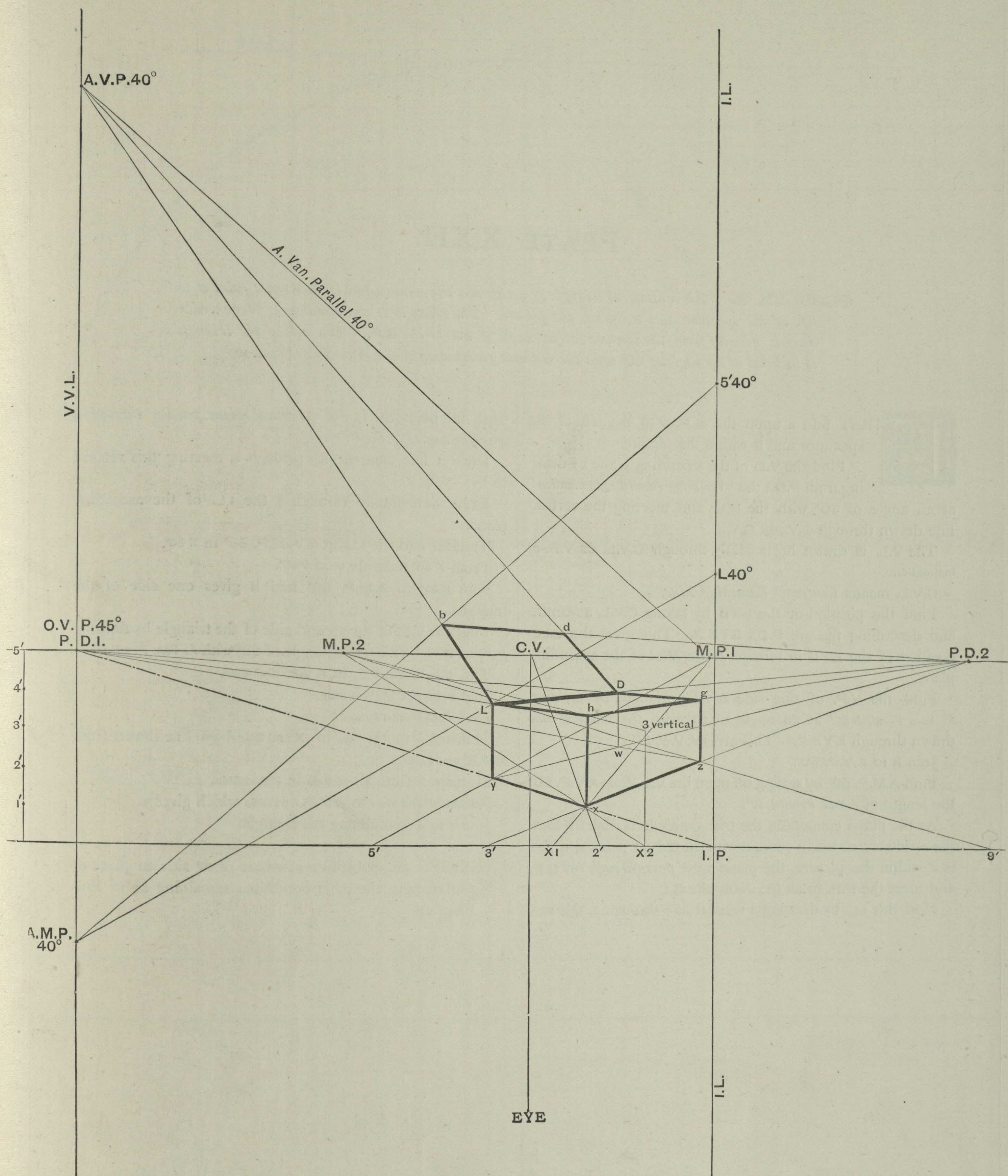

A.V.P.40°
I.L.
A. Van. Parallel 40°
V.V.L.
5′40°
L40°
O.V. P.45°
P. D.I.
M.P.2
C.V.
M. P.I
P.D.2
b
d
D
g
L
h
3 vertical
w
z
y
x
5′
4′
3′
2′
1′
5′
3′
X1
2′
X2
I. P.
9′
A.M.P.
40°
I.L.
EYE

PLATE XXII.

PROBLEM 29. *An equilateral triangle of* 9′ *side has one corner upon the* G.P. *at a point* 3′ *to left of the spectator and* 3′ *within the picture. The triangle is contained in a plane which ascends directly from the spectator at an angle of* 30° *to the* G.P.. *One side of the triangle is inclined at an angle of* 50° *with the Ground Intersection of the Ascending Plane.*

FIRST find x upon the G.P. 3′ to the left of the spectator and 3′ within the picture.

Find the V.L. of the ascending plane by drawing from P.D.1 the *"Vanishing Parallel of Inclination"* at an angle of 30° with the H.L., and meeting the vertical line drawn through C.V., in C.V.L..

The V.L. is drawn horizontally through C.V.L. or V.P. of inclination.

C.V.L. means *Centre of Vanishing Line.*

Find the position of the EYE by taking C.V.L. as centre and describing the arc P.D.1 EYE 2. This gives the exact position of the EYE in relation to the V.L. of the ascending plane.

Find the V.P. of one side of the triangle by drawing *A. Van. Parallel 50°* at an angle of *50°* with the *Directing Line* drawn through EYE 2. This gives A.V.P. 50°.

Join X to A.V.P. 50°.

Find A.M.P. 50° by setting off upon the V.L. *from* A.V.P. 50° the length of *A. Van. Parallel 50°*.

As the plane containing the triangle ascends directly from the spectator and is in contact with the G.P. at point x, which is 3′ within the picture, the plane must *pass through the* G.P. and meet the P.P. in an I.L. *below* the G.L..

Find this I.L. by drawing a vertical line through 3, this vertical line being the I.L. of a *vertical plane passing through* x *at right angles to the* P.P..

Draw a line from C.V.L. through x, meeting this vertical in I.

Draw horizontally through I, the I.L. of the ascending plane.

Transfer x to the I.L. from A.M.P. 50° in X 50°.

From X 50° measure 9′ in 9·50°.

Join 9·50° to A.M.P. 50° and it gives one side of the triangle.

Find the V.P. of the second side of the triangle by drawing *A. Van. Parallel 2nd Side* at an angle of 60° with *A. Van. Parallel 50°*.

This meets the V.L. in A.V.P. 2nd Side.

Join X to A.V.P. 2nd Side.

Find A.M.P. 2nd Side.

Transfer x to the I.L. in X 2nd Side by a line drawn from A.M.P. 2nd Side.

Measure 9′ from X 2nd Side in 9′ 2nd Side.

Return 9′ 2nd Side to A.M.P. 2nd Side which gives y.

Join y to z, completing the triangle.

EXERCISE 56. Substitute a square of 10′ side in place of the triangle, the other conditions remaining as in Problem 29.

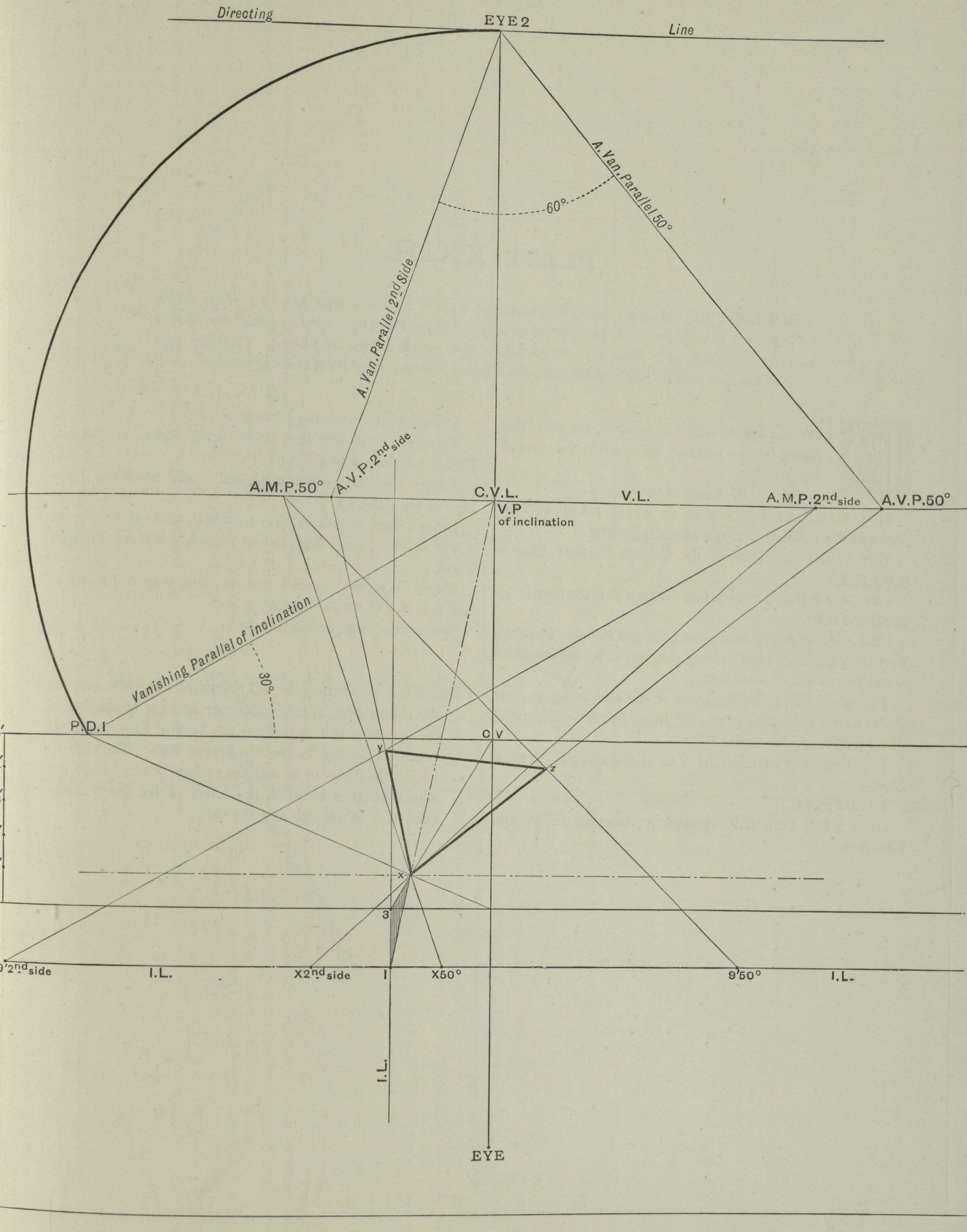

Directing
EYE 2
Line
A. Van. Parallel 50°
60°
A. Van. Parallel 2nd Side
A.V.P. 2nd side
A.M.P. 50°
C.V.L.
V.P
of inclination
V.L.
A.M.P. 2nd side
A.V.P. 50°
Vanishing Parallel of inclination
30°
P.D.I
5′
4′
3′
2′
1′
C V
y
z
x
3
9′ 2nd side
I.L.
X 2nd side
I
X 50°
9′ 50°
I.L.
I.L.
EYE

PLATE XXIII.

PROBLEM 30. *A square slab, 9′ square and 4′ thick, has one square face in a plane which ascends directly from the spectator at an angle of 30° to the G.P.. One corner of the slab is upon the G.P. at a point 4′ to the spectator's right and 4′ within the picture. One long edge of the slab is at an angle of 45° with the Ground Intersection of the Ascending Plane.*

DRAW the square face contained in the ascending plane, by the method followed in the preceding Problem.

For the thickness of the slab.

Find A.V.P.R.A.I. by drawing an *A. Van. Parallel* at *right angles* to the *inclination* of the ascending plane.

This Van. Parallel meets the Central Vertical Line in A.V.P.R.A.I..

From A.V.P.R.A.I. draw a line through X extending it towards the P.P..

Find A.M.P.R.A.I. by setting off from A.V.P.R.A.I., upon the Vertical Vanishing Line passing through C.V., the length of *A. Van. Parallel at R.A. to inclination.*

The edge giving the thickness of the slab being at right angles to the ascending plane, will be contained in a Vertical Plane perpendicular to the P.P..

This Vertical Plane has its V.L. in the Central Vertical Line.

To find its I.L..

Draw a line from C.V. through X, meeting the Picture Line in 4.

Draw the I.L. vertically through 4.

Transfer X to the P.P. from A.M.P.R.A.I. in XR.A.I. Measure from it 4′ in 4′ R.A.I..

Return 4′ R.A.I. to A.M.P.R.A.I. and it will cut off the thickness of the slab in T.

From T draw two lines, one to A.V.P. 45° L, the other to A.V.P. 45° R, to meet lines drawn from A.V.P.R.A.I. through Z and Y.

These lines give points a and b. Join b to A.V.P. 45° L, and a to A.V.P. 45° R, meeting in c.

Join c to w, completing the slab.

EXERCISE 57. Substitute an equilateral triangular slab for the square one, its nearest corner being 3′ to the left of the spectator and 4½′ beyond the Picture Line, the other conditions being as for the square slab. The slab to have a thickness of 5′ and long edges of 10′. Find the actual distance between the centre of the lower triangular face of the slab and the G.P..

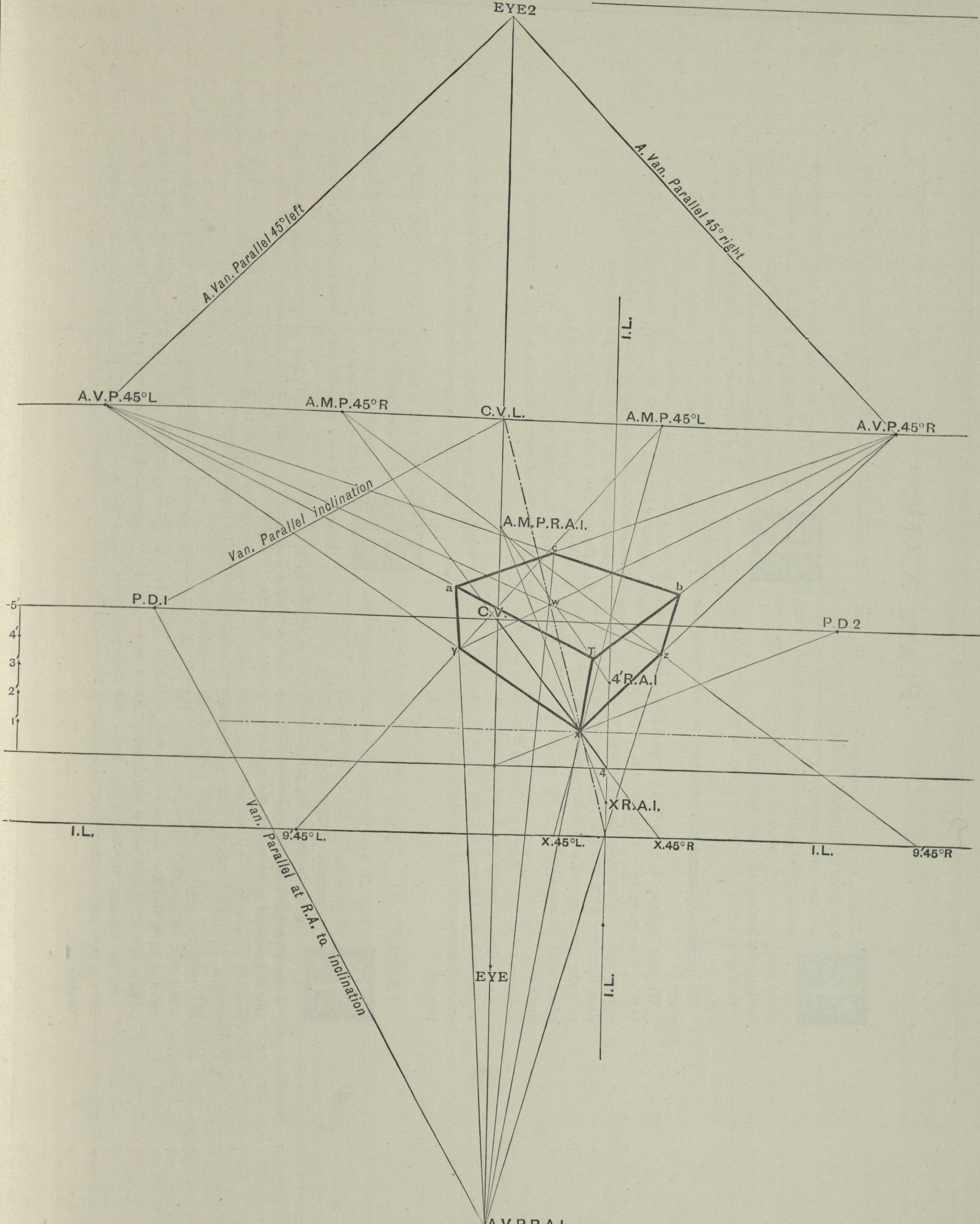

EYE2
A.Van. Parallel 45° left
A. Van. Parallel 45° right
I.L.
A.V.P.45°L
A.M.P.45°R
C.V.L.
A.M.P.45°L
A.V.P.45°R
Van. Parallel inclination
A.M.P.R.A.I.
c
a
b
w
P.D.I
C.V.
P.D 2
5'
4'
3'
2'
1'
y
T
z
4'R.A.I
x
4
XR.A.I.
I.L.
9.45°L.
X.45°L.
X.45°R
I.L.
9.45°R
Van. Parallel at R.A. to inclination
EYE
I.L.
A.V.P.R.A.I.

PLATE XXIV.

IN **Fig. I.** we have a diagram illustrating the fact that a M.P. really performs its office, that is, cuts off true measurements perspectively. A measuring point really is the *vanishing point of lines which make equal angles with the line to be measured and the* Intersecting Line *of the plane containing it.*

In **Fig. I.** a line starts from the G.L. at X, and vanishes in V.P..

We propose to measure a distance, as X Z, upon line X V.P..

We find its M.P. by Prop. VII., Page 14, and return Z to M.P.. This gives us a distance X A, in perspective, which is supposed to be equal to the real measurement X Z. If we draw from the EYE, the *Van. Parallel of M.P.*, it will be found to make equal angles with *Van. Parallel of V.P.* and the H.L. which is of course parallel to the I.L. of the plane containing X A.

Lines Z A and X A shown in Ground Plan at Z a and X a.

Z a is equal in length to Z A, and makes geometrically, equal angles with the I.L. and X a, the line to be measured. The second measurement at B is merely a repetition of the same process.

PROBLEM 31. *To use a proportional Measuring Point.*

IN **Fig. II.** we find the use of the "Proportional Measuring Point." In the case of very large measurements, which could not possibly be set off upon the paper, the proportional M.P. is useful.

Line A vanishes in V.P.. Let us measure upon Line A, any distance, as M, by means of the M.P. of line A. It gives X as the perspective measurement.

Now, by taking exactly *half* the distance between the M.P. and V.P. and setting off *half* the measurement, the result will be precisely the same. Take *half* the measurement at ½ M, and half distance from M.P. to V.P. at ½ M.P..

Join ½ M to ½ M.P. and it will intersect line A in X.

In like manner, by taking *one fourth* the distance from V.P. to M.P. and *one fourth* the measurement, we obtain the same result.

PROBLEM 32. *To draw a line upon the G.P. when the V.P. is inaccessible.*

IT sometimes happens that a V.P. falls very far out of the paper, so that it is impossible to fix its exact position. In **Fig. III.** a method is given by means of which a line may be drawn in its proper direction even though the V.P. is inaccessible. It is required to draw a line from point A to a V.P. on the right, which is inaccessible, but its angle is known and indicated by its *Vanishing Parallel.*

Draw a straight line *at any angle*, meeting the Vanishing Line and the Van. Parallel in 1 and 2. Join 1 and 2 to A.

Draw line 3 4 at any distance, *parallel to* 1 2.

Draw a line from 3 *parallel* to 1 A.

Draw a line from 4 *parallel* to 2 A.

These lines meet in X.

A line drawn from A through X is the line required.

PROBLEM 33. *To find the Measuring Point of an inaccessible Vanishing Point.*

IN **Fig. IV.** a method is given to find the M.P. of an inaccessible V.P..

Assume any point A upon the *Vanishing Parallel of the inaccessible V.P.*

Draw a line from A *parallel* to the V.L., which is in this case the H.L..

From A as centre, A EYE as radius, describe an arc meeting the line drawn from A, in B.

Draw a line from EYE through B, and it will meet the V.L. in the required M.P..

EXERCISE 58. Find point A, 50′ to the left, 100′ within the picture, and 70′ above the G.P.. From A draw a horizontal line A B, 100′ long and vanishing to the right at an angle of 10° with the P.P..

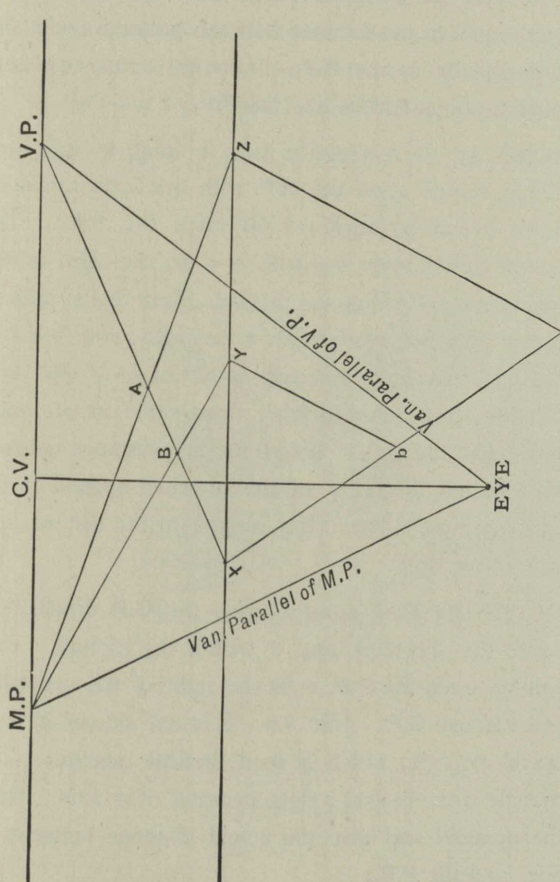


Fig.I.

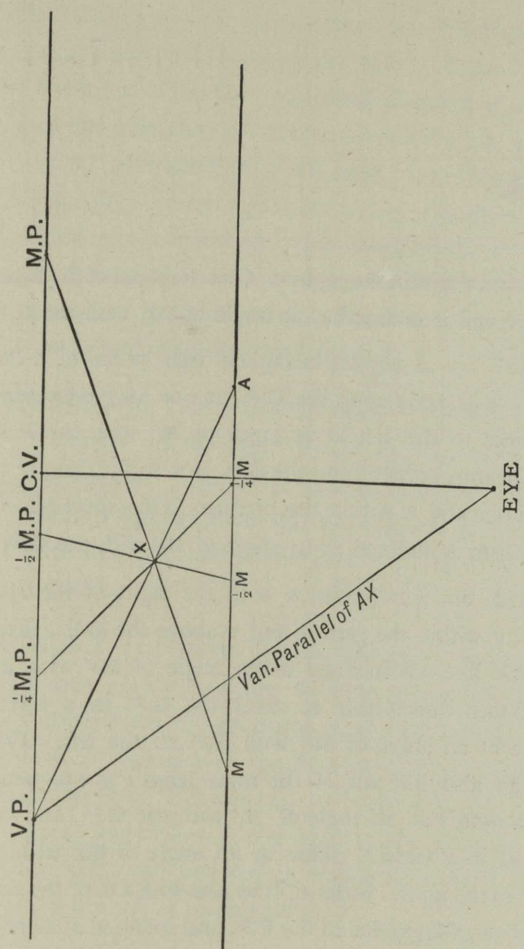


Fig.II.

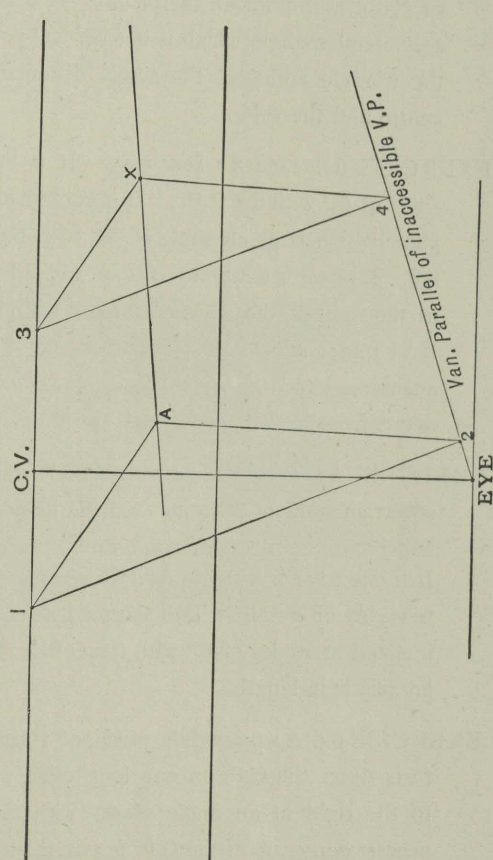


Fig.III.

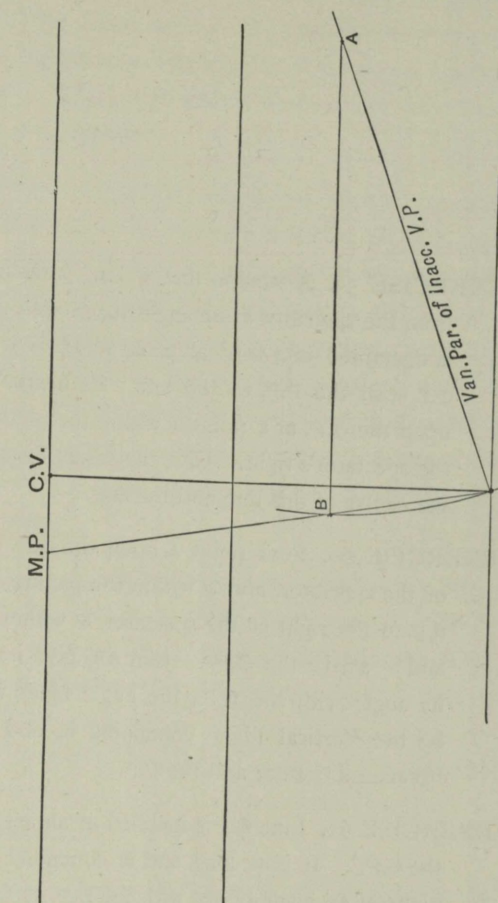


Fig.IV.

EXERCISES.

EXERCISE 59. A straight line 12′ long is inclined upwards from the spectator at an angle of 55° with the G.P.. It is contained in a vertical plane which is at an angle of 30° with the P.P. to the left. Its nearer extremity is upon the G.P. at a point 3′ within the picture and 5′ on the spectator's right. Give the actual distance between the centre of this line and the G.P..

EXERCISE 60. Find point A upon the G.P. 3′ to the left of the spectator, and 3′ within the picture. Find point B, 5′ to the right of the spectator, 5′ within the picture, and 5′ above the G.P.. Join AB, give its true length, its angle with the G.P., the angle made with the P.P. by the Vertical Plane containing it, and the distance between its centre and the P.P..

EXERCISE 61. Line AB is inclined at an angle of 10° with the G.P.. It is 11′ long and is contained in a vertical plane at an angle of 45° with the P.P. to the right. Its nearer extremity is upon the G.P. directly opposite the spectator and 4′ within the picture. It is one side of an equilateral triangle which is in a vertical position. Draw the triangle and give the actual distance between its centre and the P.P..

EXERCISE 62. Line XZ is 9′ long. It is inclined at an angle of 65° with the G.P.. It is contained in a vertical plane which is at an angle of 35° with the P.P. to the left. Its nearer extremity is upon the G.P. at a point 2′ to the right of the spectator and 6′ within the picture. It is one diagonal of a square which is in a vertical position. Draw the square and give the actual distance between its furthest corner and the P.P..

EXERCISE 63. Line AB is upon the G.P. vanishing to the left at an angle of 30° with the P.P., its nearer extremity being 3′ to the spectator's right and 6′ within the picture. It is one edge of a right prism whose ends are equilateral triangles of 6′ side. Two short edges of the prism are inclined at angles of 15° with the G.P.. The prism has an axis 9′ in length.

EXERCISE 64. A square right prism, of 7′ base and 10′ axis, rests upon the G.P. on one long edge which vanishes to the right at an angle of 40° with the P.P.. The nearest corner upon the G.P. is 4′ to the left of the spectator and 7′ within the picture. One diagonal of the nearer square end is inclined at an angle of 70° with the G.P..

EXERCISE 65. A regular hexagonal right prism of 5′ base and 11′ axis, rests upon the G.P. on one long edge which vanishes to the left at an angle of 25° with the P.P.. The nearest corner upon the G.P. is 5′ to the right of the spectator and 8′ within the picture. One short edge of the prism is inclined at an angle of 10° with the G.P..

EXERCISE 66. Find point A 6′ to the right of the spectator, 6′ within the picture and 6′ above the G.P.. From A draw line AB inclined at an angle of 30° with, and vanishing downwards to meet the G.P. in a vertical plane at an angle of 30° with P.P. to the left. From A draw also line AC, of the same length as AB, vanishing upwards at an angle of 75° with the G.P., and contained in a vertical plane at an angle of 60° with the P.P. to the right. From A draw also line AD, of the same length as AB, parallel to the P.P. and inclined at an angle of 10° with the G.P.. From A draw also line AF of a length equal to the distance between points B and D, and perpendicular to the P.P.. Give the actual distances between points B,C,D,F, and the P.P..

EXERCISE 67. A rectangular box, 12′ long, 9′ wide, and 6′ high, stands upon the G.P. with one side vanishing to the left at an angle of 60° with the P.P.. The nearest corner upon the G.P. is 4′ to the right of the spectator and 7′ within the picture. Show the lid opened in four different positions; 1st vertically, 2nd horizontally, 3d vanishing upwards at an angle of 30° with the G.P., and 4th vanishing downwards at an angle of 45° with the G.P.. Give the actual distance between the P.P. and the centre of the lid when opened in the third position. The lid is hinged to the distant long edge of the box.

EXERCISE 68. Find point Z upon the G.P. directly opposite the spectator, and 6′ within the picture. Find point X upon the P.P. 8′ to the right of the spectator, and 8′ above G.P.. Join XZ. It is one side of an equilateral triangle, which is in a vertical position. The triangle is the base of a right pyramid of 15′ axis. Draw the pyramid and give the actual distance between its apex and the P.P..

EXERCISE 69. Find point H upon the P.P. 2′ to the spectator's left and 7′ above the G.P.. From H draw line H K to meet the G.P., vanishing downwards at an angle of 50° with the G.P. and in a vertical plane at an angle of 40° with the P.P. to the right. This line is one long edge of a square right pyramid of 7′ base, H being the apex of the pyramid. One diagonal of the pyramid's base is in the vertical plane which contains line H K. Draw the pyramid and give the actual distance between the centre of its base and the P.P..

EXERCISE 70. A circle 8′ in diameter touches the G.P. at a point 4′ to the right of the spectator and 4′ within the picture. The circle is contained in a plane which ascends directly from the spectator at an angle of 25° with the G.P., and is one face of a slab 4′ in thickness. Draw the slab.

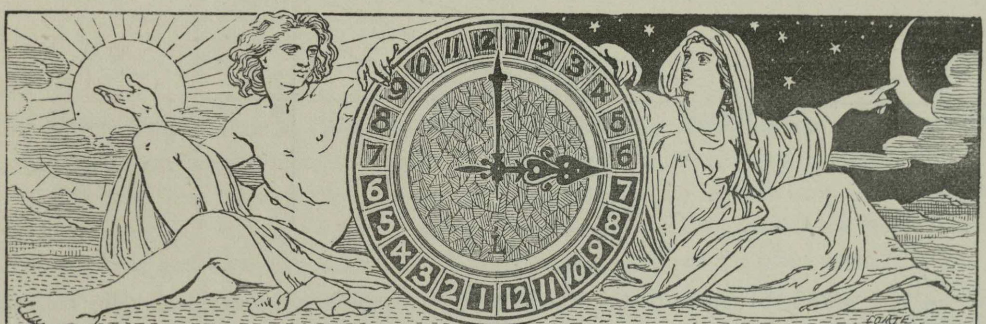
EXERCISE 71. An equilateral, triangular, right pyramid of 7′ base and 11′ axis has its base in a plane ascending directly from the spectator at an angle of 40° with the G.P.. One edge of the base is inclined to the right at an angle of 20° with the Ground Intersection of the Ascending Plane. The apex of the pyramid touches the P.P., and one corner touches the G.P. at a point 2′ to the left of the spectator.

EXERCISE 72. A cube of 6′ edge has one face in a plane which ascends directly from the spectator at an angle of 45° with the P.P.. One edge of the cube is at an angle of 45° with the Ground Intersection of the Ascending Plane. One corner of the cube touches the P.P. at a point 4′ on the spectator's right. One corner touches the G.P.. Upon the nearest face of the cube show a circle 6′ in diameter.

EXERCISE 73. Line A B is 10′ long and vanishes upwards from the spectator at an angle of 45° to the G.P., and is in a vertical plane at an angle of 45° with the P.P. to the left. Its nearer extremity is upon the G.P. at a point 4′ to the right and 3′ within the picture. A B is one diagonal of a square, the other diagonal of which is horizontal. The square is the base of a right pyramid of 10′ axis. Give the actual distance between the apex of this pyramid and a point upon the G.L. directly opposite the spectator.

EXERCISE 74. A regular pentagonal slab having edges of 7′ has one face in a plane which ascends directly from the spectator at an angle of 35° with the G.P.. The nearest corner of the slab is upon the P.P. directly opposite the spectator, another corner being upon the G.P.. One edge of the slab is at an angle of 35° with the Ground Intersection of the Ascending Plane. Thickness of slab 5′.

EXERCISE 75. A square has sides of 100′. It is upon the G.P. with one side vanishing to the left at an angle of 10° with the P.P.. The nearest corner is 5′ to the left of the spectator and 80′ within the picture. This square is the base of a right pyramid having an axis of 70′. Draw the pyramid.



THEORY.

ONE of the first things stated in the beginning of our study was that the Horizontal Line is the Vanishing Line of the Ground Plane (see page 10), that is, the Ground Plane infinitely extended, could only appear to reach the Horizontal Line, and as no part could appear above the Horizontal Line, the plane is said to vanish there. It is indeed a truth that *all* horizontal planes vanish in the Horizontal Line.

Referring to Fig. 23, we find the *Picture Plane,* the Eye, and three horizontal planes, marked H I K, 2, and X 3 Z, the first being the *Ground Plane.* These are all shown in side elevation.

Three points, A, B, and C, are marked upon the *Ground Plane,* and three points, D, E, and F, are marked upon plane X 3 Z. Drawing the rays from points A, B, and C, to the Eye, we find the portion of the *Ground Plane* marked A represented

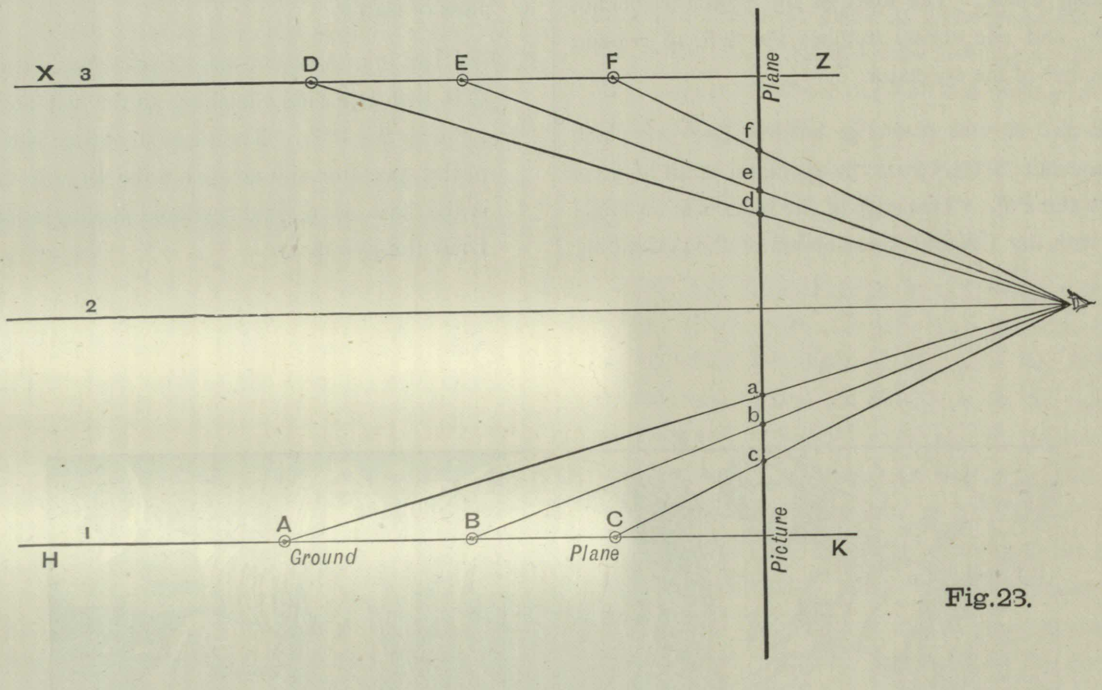


Fig. 23.

upon the *Picture Plane* at a, of B at b, and of C at c, the representation of the *furthest* point being *highest* upon the Picture Plane. Thus, as the *Ground Plane* retires, it appears to approach the central position opposite the eye (see page 10). The same is also equally true of plane X 3 Z above the level of the Eye, points D, E, and F having their representations upon the Picture Plane in d, e, and f, while the plane marked 2 is seen merely as a line at the level of the spectator's Eye. Thus the horizontal plane *below* the level of the eye appears to *ascend*, while the horizontal plane *above* the level of the eye appears to *descend*, both of them approaching the level of the Eye. If the two planes are produced infinitely and the same process repeated, viz. points marked on each plane, and visual rays drawn from the points to the eye, as the points recede from the Picture Plane the visual rays will more closely approach the central position; and, could the planes be produced far enough, the visual rays would ultimately appear to coincide with

the Central Visual Ray. Therefore the Vanishing Line of all horizontal planes will be the Horizontal Line.

In like manner all Vertical Planes at right angles to the Picture Plane appear to vanish in a Vertical line passing through the Centre of Vision.

Referring again to Fig. 23, let us consider the Picture Plane, Eye, and the three planes to be viewed, in *plan*, the three planes now becoming vertical planes at right angles to the Picture Plane. By going through the same process as before, it will be seen that as the planes X 3 Z and H I K retire, they appear to approach the central position.

We may then consider the theoretical truth established, that *all horizontal planes appear to vanish in the Horizontal Line*, also that *all vertical planes at right angles to the Picture Plane vanish in the Central Vertical Line.*

It may also be noted that all planes (whether vertical or inclined) at right angles to the Picture Plane, will have their Vanishing Lines passing through the Centre of Vision.

Turning to page 12, our first proposition says that all real measurements must be made upon the Picture Plane. The truth of this will be evident on reference to Fig. IV., Plate I., from which we may deduce the fact that a measurement of say 1′ at a distance of 3′ from the Picture Plane would *appear* less than its real size. It would therefore give a false result if we made a real measurement, within the picture.

Our second proposition (see page 12) states that lines which are at right angles to the Picture Plane vanish in the Centre of Vision.

In Fig. 24 we have a ground plan of the *Picture Plane*, the position of the spectator, and two lines A B and C D, at

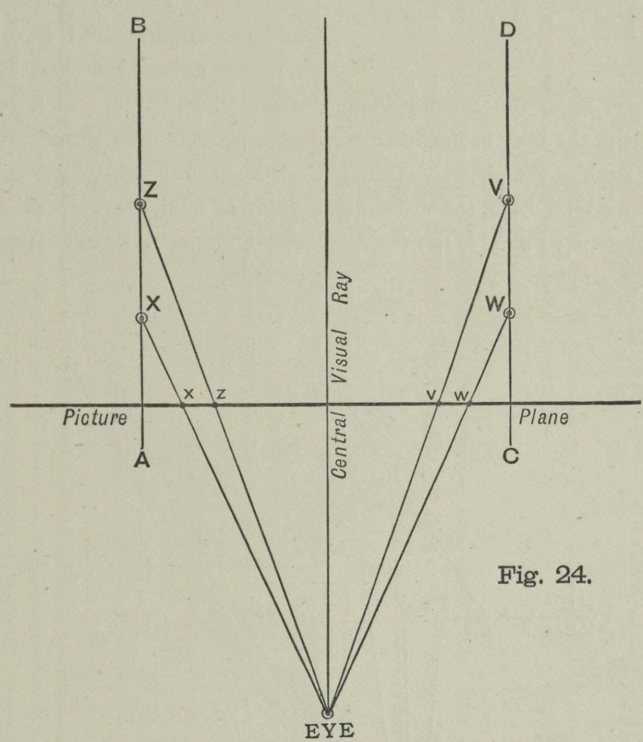


Fig. 24.

right angles to the Picture Plane. Upon line A B, two points are marked X and Z. Upon line C D also, two points are marked V and W. The rays from X and Z drawn to the eye cut the *Picture Plane* in x and z. The rays from V and W drawn to the eye cut the *Picture Plane* in v and w. The *Central Visual Ray* is at right angles to the Picture Plane. Therefore as the representations of lines A B and C D are seen to continually approach the *Central Visual Ray*, which passes through the Centre of Vision, but both lines being parallel to the *Central Visual Ray*, the rays drawn from the lines to the eye can never go beyond it, lines at right angles to the *Picture Plane* will have their Vanishing Point in the Centre of Vision.

Our third proposition (see page 12) says that the Point of Distance is the Measuring Point for lines which vanish in the Centre of Vision. Turning to Plate XXIV. we find it proved that a Measuring Point is really the Vanishing Point of lines which make equal angles with the line to be measured and the intersecting line of the plane containing it. By joining the eye to the Point of Distance we find that its Vanishing Parallel is at an angle of 45° with the Picture Plane. Then lines drawn to the Point of Distance are at angles of 45° with the Ground Line, and consequently also at angles of 45° with lines at right angles to the Picture Plane, therefore lines which vanish in the Centre of Vision are measured by lines which vanish in the Point of Distance.

Our fourth proposition (see page 12) says that any point upon the Horizontal Line may be used as a measuring point for

vertical lines. The reason for this is that all vertical planes passing through and limited by the same vertical line must be equal, and coming in contact with the Picture Plane will have vertical intersections of equal magnitudes (see Fig. 25). X Z is a vertical line, X being upon the Ground Plane. A B, V.I., and C D, V.2., are two vertical planes passing through

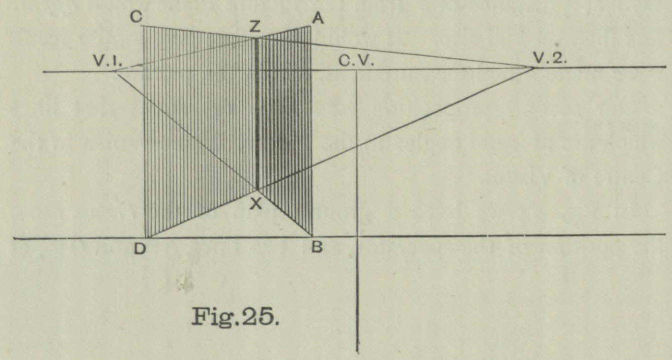


Fig.25.

X Z. Their vertical intersections upon the Picture Plane A B and C D, which correspond with the lines of heights used in working, are of equal length, each of the planes vanishing to different points upon the Horizontal Line, the vanishing points V.I. and V.2. being the Measuring Points of the vertical line X Z.

This will also be found to bear out the fact referred to in Plate XXIV., page 64, viz. that a Measuring Point is a Vanishing Point of lines which make equal angles with the line to be measured and the Intersecting Line of the plane containing it. Thus, in Fig. 25, if we take plane A B, V.I., A B is the Intersecting Line. It is vertical. X Z is the line to be measured. It is also vertical. Lines B V.I., A V.I. are horizontal, therefore they make equal angles with the line to be measured and the Intersecting Line of the plane containing it.

Our fifth proposition (see page 12) states that when the extremity of the line to be measured is within the picture, it must be transferred to the Picture Plane from its measuring point. As Proposition I. shows that all real measurements must be made upon the Picture Plane it is clear that the extremity must be transferred to the Picture Plane.

That it must be transferred from its Measuring Point will be evident by a reference to the explanation of Proposition 3 and an examination of Fig. 1, Plate XXIV., in which figure we have a line X B A upon the Ground Plane, point B being within the picture. If it be required to measure a length from point B upon the line X B A, the point B must be transferred to the Picture Line by a line drawn from its Measuring Point, as a line drawn from any other point would not make equal angles with the Picture Line and the line to be measured.

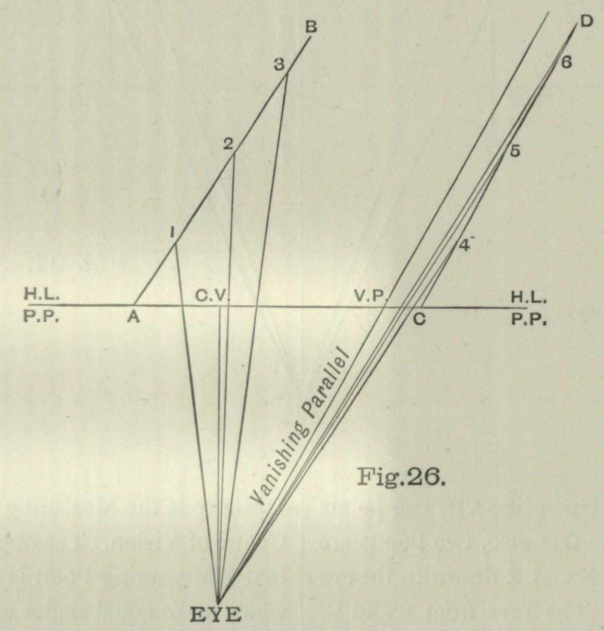


Fig.26.

Our sixth proposition (see page 14) says that to find the Vanishing Point of a horizontal line, a line must be drawn from the eye to meet the Horizontal Line, parallel to the line whose Vanishing Point is required. On reference to Fig. 24 it will be seen that the *Central Visual Ray* is the Vanishing Parallel of lines at right angles to the *Picture Plane.* In Fig. 26 we have the same principle illustrated by means of a line inclined to the Picture Plane. The Picture Plane, two lines A B and C D, their *Vanishing Parallel,* and the EYE are seen in *plan.* In this figure as in Fig. 1, Plate II., the H.L. represents the Picture Plane seen in plan. A careful comparison of this figure with Fig. 24 will be all required to clearly show the truth of the proposition.

Our seventh proposition (see page 14) is to the effect that to find a Measuring Point we must set off upon the Vanishing Line of a plane, the length of the Vanishing Parallel. As this process gives upon the Vanishing Line a Vanishing Point of lines making equal angles with the line to be

measured to the Intersecting Line of the plane containing it, and has been explained (see Fig. 1, Plate XXIV.) in the foregoing demonstrations, further explanation will be unnecessary.

Our eighth proposition (see page 14) states that lines which are parallel vanish to the same point. The truth of this proposition, if not already realised, will be made manifest by a reference to Fig. 24.

Our ninth proposition (see page 50) says that to find the Vanishing Point of an inclined line we must draw from the Original Measuring Point to meet the Vertical Vanishing Line, a line parallel to the line whose Vanishing Point is required.

We have seen that Vertical planes at right angles to the Picture Plane have their Vanishing Lines passing vertically through the Centre of Vision. In like manner the Vertical Vanishing Lines of Vertical Planes making other angles with the Picture Plane will be on the right or left of the Centre of Vision as the case may be.

Thus, if a line incline upwards or downwards to the right, the Vertical Vanishing Line upon which it will vanish will be on the right; and if the line incline to the left, the Vertical Vanishing Line will be on the left.

This will be easily seen by reference to Figs. 23 and 26.

In Fig. 27 we have the Picture Plane, XYZ, and the Ground Plane HKYZ. A triangle ABC stands upon the Ground Plane in a vertical position, side AB being inclined to the Ground Plane.

The eye is shown in its real position in front of the Picture Plane, and the *V.V.L.* of the *Vertical Plane* containing *AB*, with its *Horizontal Vanishing Parallel* giving the *O.V.P.* upon the Horizontal Line. The real Vanishing Parallel of AB is drawn actually parallel to AB.

The distance between *O.V.P. of Vertical Plane containing AB*

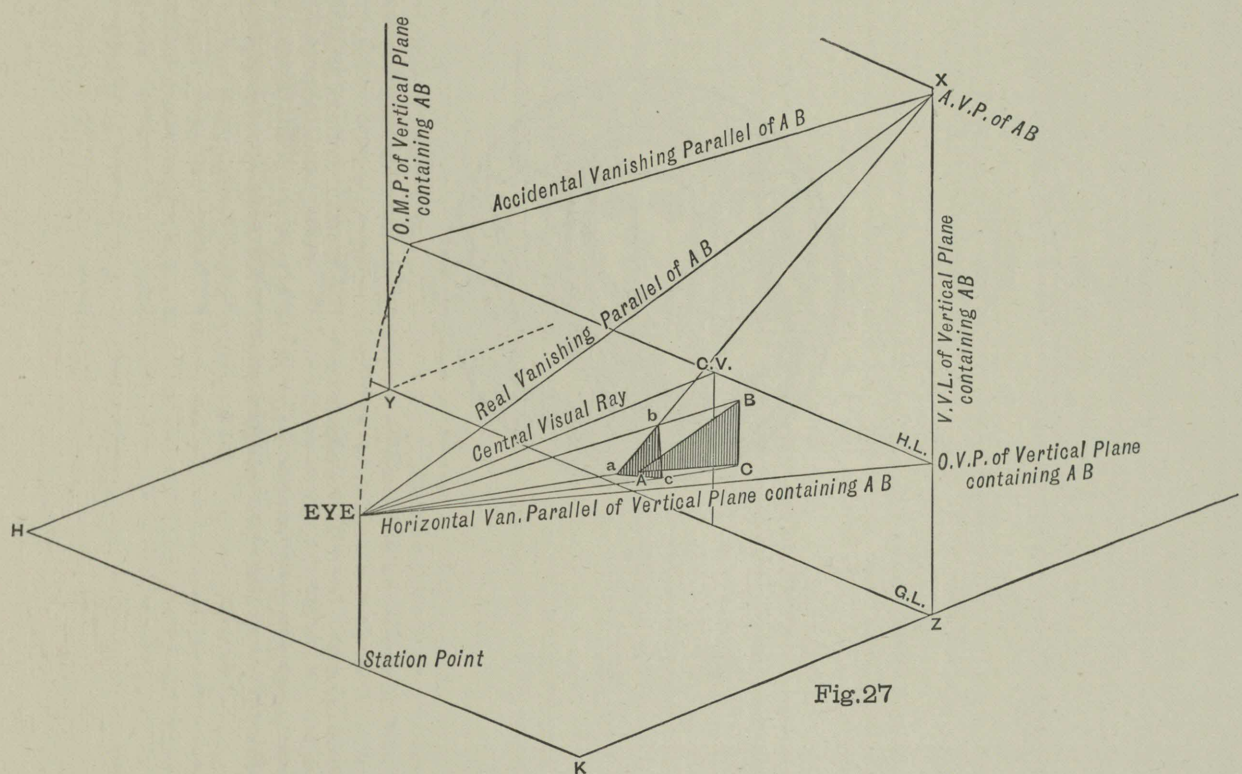


Fig. 27

upon the Picture Plane and *O.M.P. of Vertical Plane containing AB* upon the Picture Plane is, in actual problem made *equal to the distance between the eye and O.V.P. upon the Picture Plane.*

Therefore a line drawn from *O.M.P. making the same angle with the Horizontal Line as is made by the real Vanishing Parallel and the Horizontal Vanishing Parallel,* will give the true position of the *A.V.P.*.

The tenth proposition (see page 50) refers to the position of an Accidental Measuring Point. This will need no explanation as the principles governing it have been fully discussed in the preceding demonstrations.

The Vanishing Line of a Direct Ascending Plane is determined by the intersection with the Picture Plane of a plane passing through the spectator's eye at an angle with the Ground Plane equal to that made by the Direct Ascending Plane. In a similar way the Horizontal Line is the intersection of the Picture Plane with a plane passing *horizontally* through the spectator's eye.

We are now conversant with the elementary points connected with the theory of linear perspective, and should be able to easily grasp the principles governing shadows and reflections, to be considered in our next Part.

POYNTER'S SOUTH KENSINGTON DRAWING-BOOK.

ELEMENTARY PERSPECTIVE DRAWING.

BY

S. J. CARTLIDGE, F.R.Hist.S.

PART IV.

PLATE XXV.

IN this, the fourth division of our book, we commence the consideration of the method of projecting shadows.

If we place a lighted lamp upon the surface of a table, and hold the hand between the table and the lamp, with the palm facing the lamp, the palm thus receiving the full light of the lamp, will be said to be "*in light;*" and the back of the hand, (not receiving direct light from the lamp) "*in shade.*"

If the hand is held near the table we see a darkened portion of the surface of the table, caused by the interception of the hand between the surface of the table and the light. This darkened portion is called the "*shadow.*"

This "*shadow*" varies in size, shape, and strength, according to the relative positions of the hand, the table, and the light.

If the surface receiving the "*shadow*" be a plane, it is called the "*Plane of Shadow.*"

To determine the size and shape of any shadow, we must know—

1st. *The position of the light.*

2nd. *The position of the surface receiving the Shadow.*

3rd. *The position, shape, and size of the object causing the shadow.*

PROBLEM 34, Fig. I. *To project the shadow of a vertical rod 3′ high, resting upon the Ground Plane. The rod is 2′ to the right of the spectator, and 6′ within the picture. The shadow of the rod is required upon the Ground Plane when the position of the light is 5′ to the left of the spectator, 3′ within the picture, and 8′ above the Ground Plane.*

THE rod is shown at R r, r being upon the G.P..

The position of the light is shown at L, 5′ to the left of the spectator, 3′ within the picture and 8′ above the G.P..

The method is as follows—

The lower extremity, r, is *upon the Plane of Shadow*, so that it will be unnecessary to find its projection.

We must, then, find the shadow of the upper extremity R, which is 3′ above the G.P..

Drop R vertically to the Plane of Shadow in r.

Drop the light L vertically to the Plane of Shadow in l.

Join l r producing the double chain dotted line indefinitely within the picture.

This double chain dotted line is the *Ground Intersection of a Vertical Plane which passes through the light and the point* R.

Draw a ray of light from L through R meeting the double chain line in S.

S is the shadow of R.

Join S r, it is the shadow of the rod R r.

PROBLEM 35, Fig. II. *To project the shadow of a rectangular slab upon the Ground Plane when the light is 7′ to the right of the spectator and 11′ above the Ground Plane.*

THE method is exactly as for the rod in the preceding problem.

We will first find the shadow of corner A.

A is dropped vertically to the G.P. in a.

The light L is dropped vertically to the G.P. in l.

Draw the ground intersection through l a, and draw the ray from L through A to meet the intersection in 1.

1 is the shadow of A.

Join 1 a.

1 a is the shadow of A a.

B is dropped vertically to the G.P. in b.

Draw the ground intersection from l through b.

Draw the ray from L through B, meeting the ground intersection in 3 which is the shadow of B.

Join 1 3, it is the shadow of A B.

C is dropped vertically to the G.P. in c.

Draw the ground intersection from l through c.

Draw the ray from L through C, meeting the ground intersection in 5 which is the shadow of C.

Join 3 5, it is the shadow of B C.

Join 5 c, it is the shadow of C c.

The *shadow* is indicated by parallel horizontal lines.

The *shade* is indicated by parallel vertical lines.

EXERCISE 76. Cast the shadow of the vertical line H K in Exercise 15, page 22, upon the Ground Plane when the light is 7′ to the left, 5′ within the picture, and 17′ above the Ground Plane.

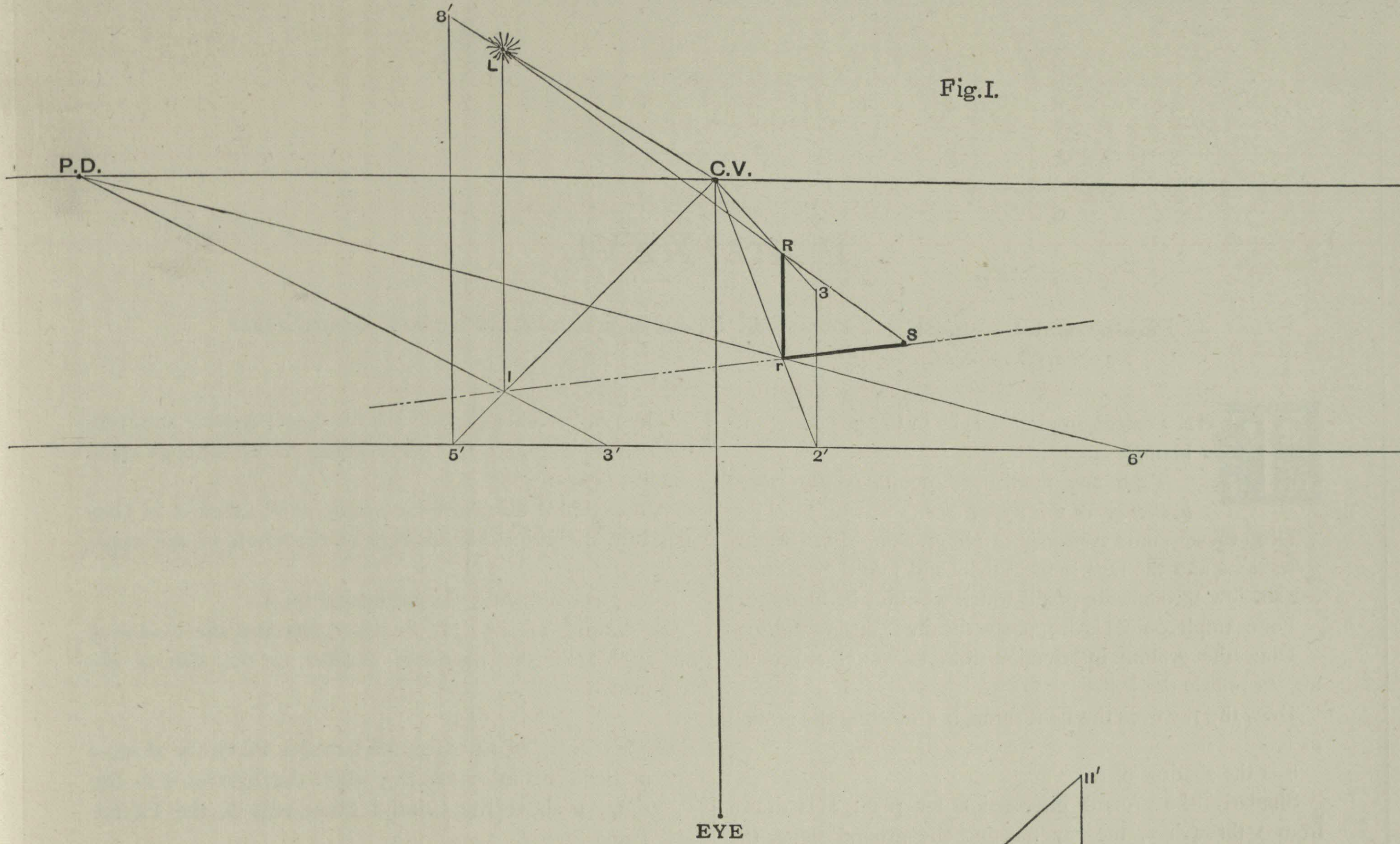


Fig.I.

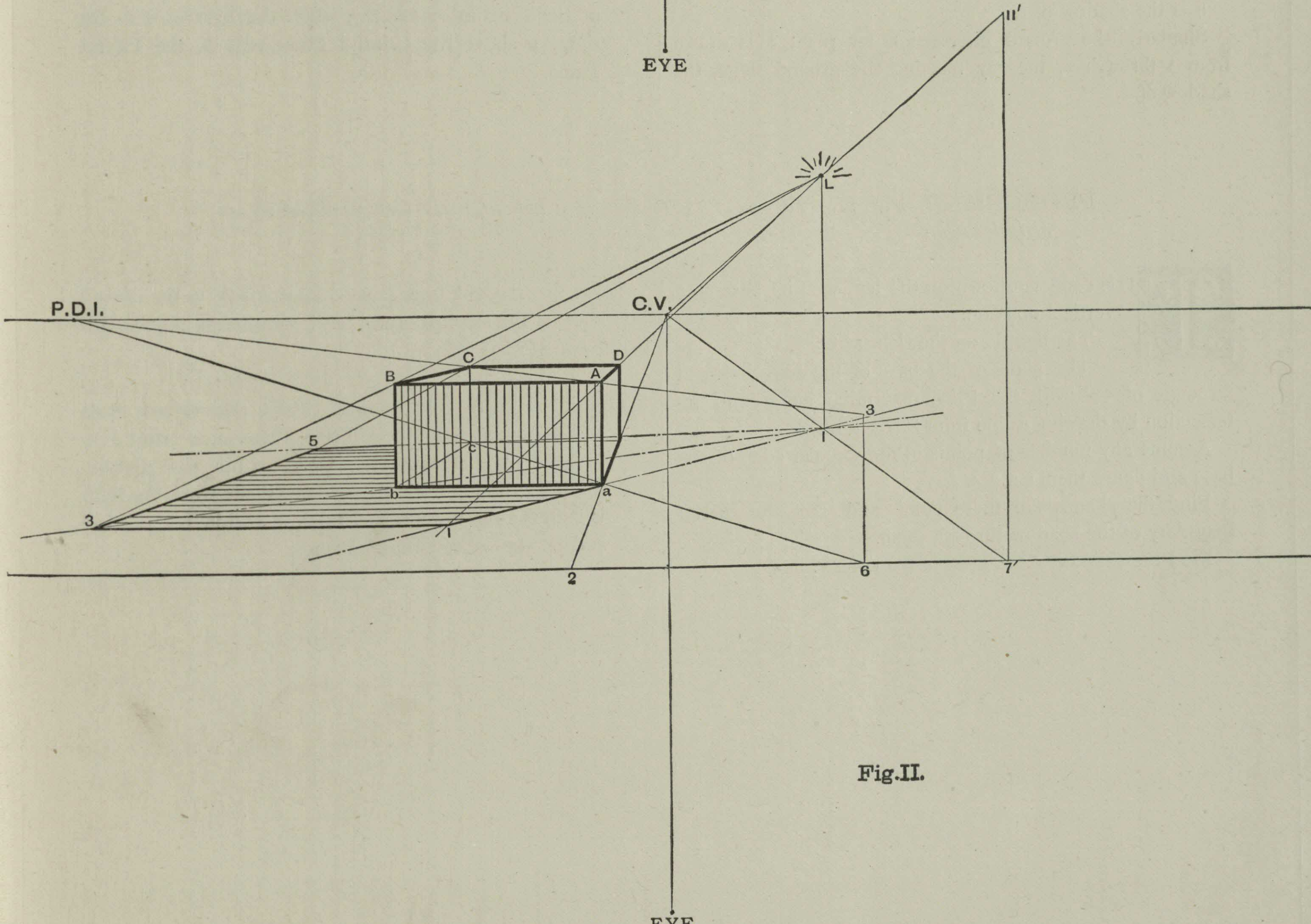


Fig.II.

PLATE XXVI.

PROBLEM 36, Fig. I. *To project the shadow of a circular slab upon the Ground Plane by an artificial light.*

THE process employed is as in the preceding problem.

Take any number of points *in the curved boundary* of the upper face.

Drop these points *vertically to the ground.* Draw the intersections, and the rays to meet them and join the outline of the shadow through the points which will thus be found.

For example, point I is dropped vertically to the ground in a.

Draw the ground intersection from I through a, producing it towards the P.P..

Draw the ray from the light through I, meeting the ground in II.

II is the shadow of I.

Similarly, the ground intersection for point 3 is drawn from I through b, the ray meeting the ground in I3, the shadow of 3.

Thus any number of points in the object may be assumed, their shadows found, the curve being traced through these shadow points.

Although not absolutely necessary in all cases, it is best, as a rule, to project the shadow of the whole of the upper face.

This gives a shadow of an elliptical form.

By drawing *tangents* to the *lower face* and the *shadow of the upper face* the complete shadow of the slab is determined.

EXERCISE 77. Cast upon the Ground Plane the shadow of the cone in Problem 7, when the light is 6′ to the right, 9′ above the Ground Plane and *in* the Picture Plane.

PROBLEM 37, Fig. II. *To project the shadow of a cone upon the Ground Plane by an artificial light.*

THE cone is shown on the left, with its base in a vertical position.

The light is on the right at L.

The plane of the base of the cone being at an angle of 45° with the P.P., determine its ground intersection by drawing a line from P.D.I through a.

Assume any number of points in the circular edge of the base and figure them I, 2, 3, 4, &c.

Find the shadows of these points and trace the curved boundary of the shadow through them.

Find the shadow of the apex of the cone V, in v.

Draw two straight lines from v, *tangentially* to the curved boundary of the shadow of the base, completing the shadow of the cone.

EXERCISE 78. A right cone of 7′ base and 4½′ axis rests with its apex upon the Ground Plane at a point 3′ to the left of the spectator and 7′ within the picture. Show its shadow upon the Ground Plane when the light is 5′ to the right of the spectator, 4′ within the picture, and I2′ above the Ground Plane.

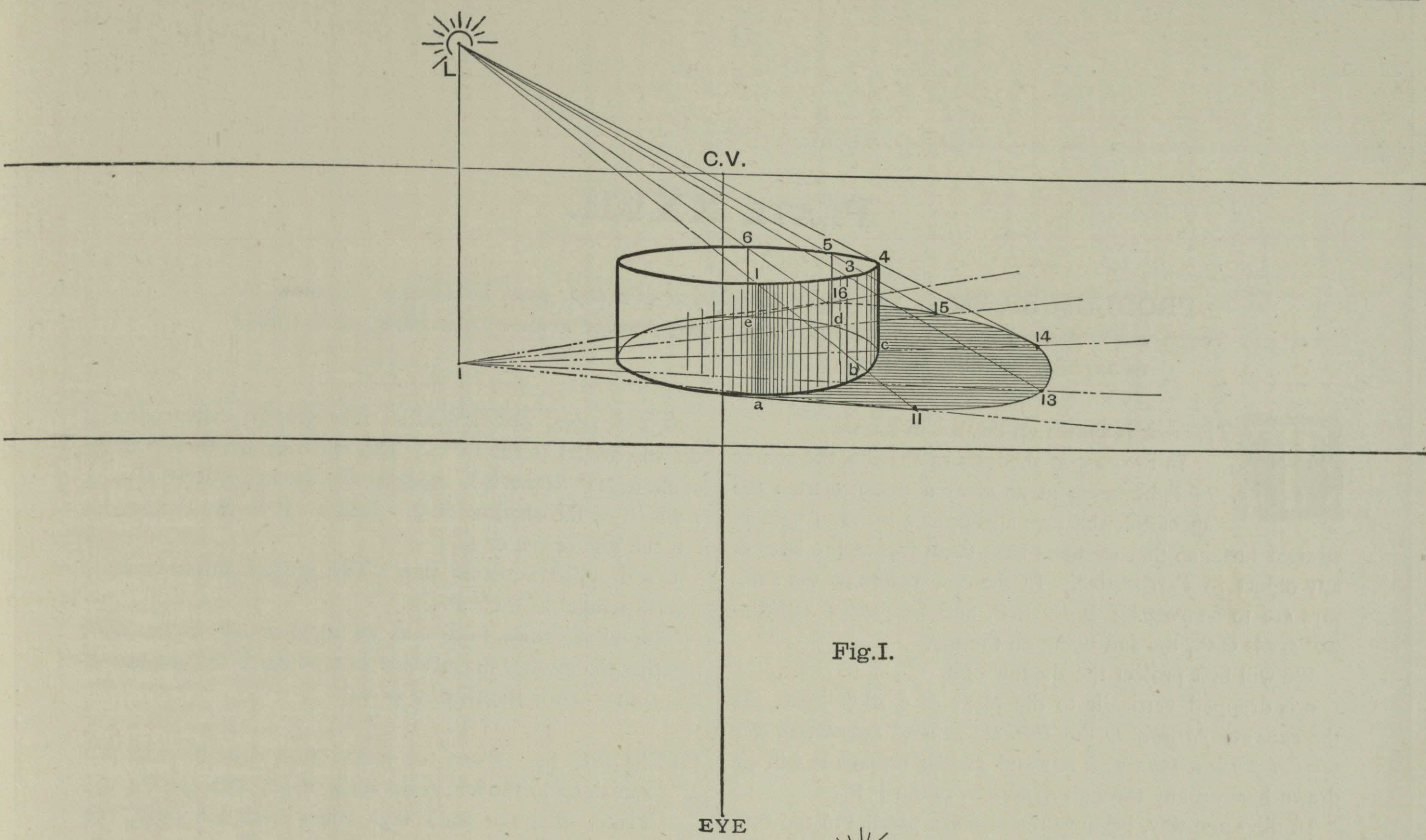

L
C.V.
6
5
4
3
1
16
15
14
13
11
e
d
c
b
a
EYE

Fig.I.

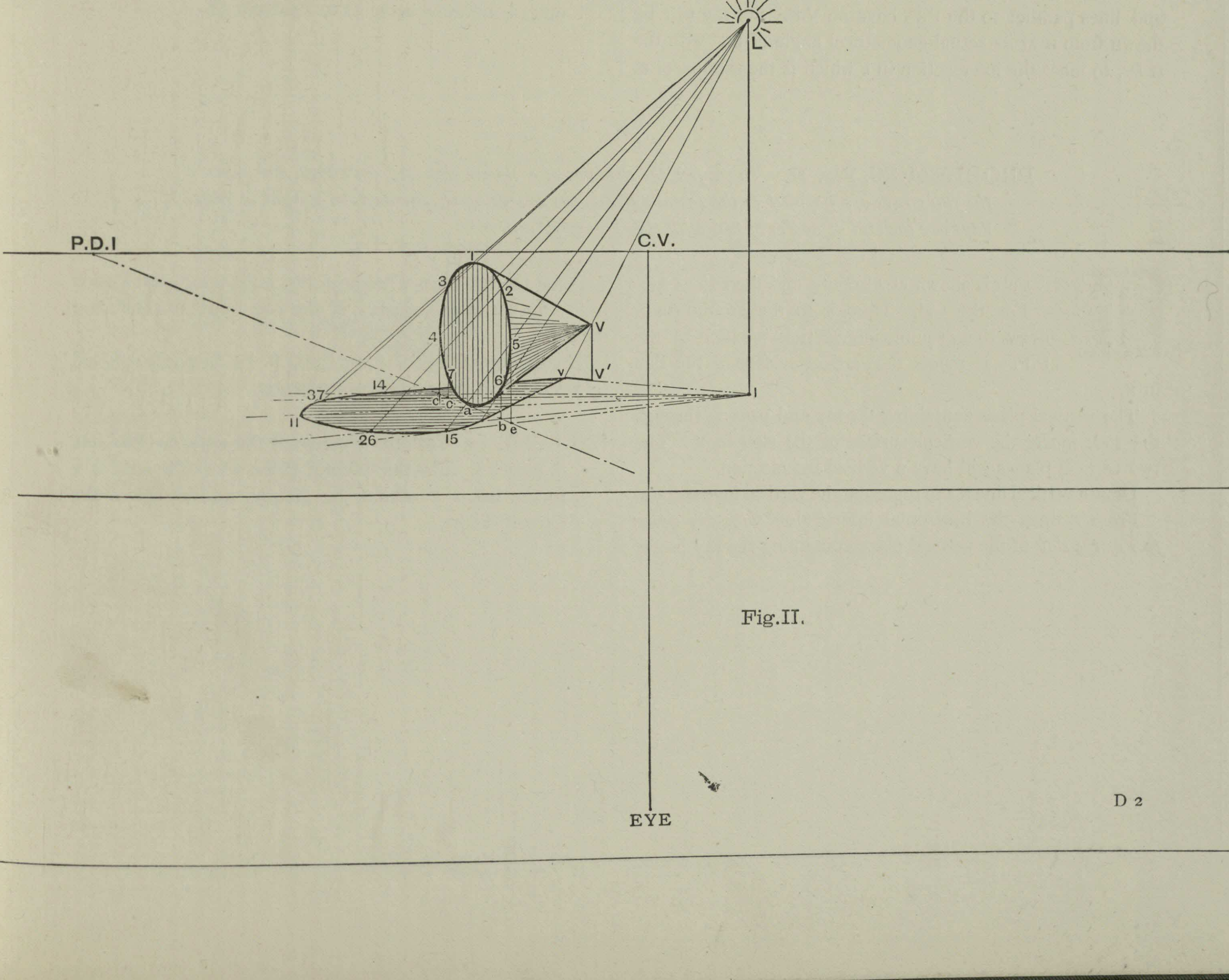

L
P.D.I
C.V.
3
2
4
5
V
V′
v
7
6
37
14
d
c
a
11
26
15
b
e
EYE

Fig.II.

PLATE XXVII.

PROBLEM 38, Fig. I. *To project the shadow of a cross upon the Ground Plane by the sun's rays. The sun is on the left, its rays being parallel to the Picture Plane, and inclined at angles of* 30° *with the Ground Plane.*

THE cross is shown on the left in R C Oc.

In the case of shadows by the sun, the source of light being at so great a distance from the spectator, the rays appear as a series of parallel straight lines, so that we must treat them just as the lines of any object in Perspective. In the case before us the sun's rays are to be *parallel to the* P.P. and *inclined at angles of* 30° *to the* G.P., the sun being on the *left.*

We will first project the shadow of R.

R is dropped vertically to the plane of shadow in r. As the *rays* are *parallel to* the P.P. *the ground intersection of a vertical plane containing a ray* and *passing through* R, will be drawn horizontally through r, *parallel to* the P.P..

In the same way, because the rays are parallel to the P.P. and lines parallel to the P.P. have no V.P., the ray will be drawn from R at its actual geometrical angle of 30° with the G.P., to meet the intersection in I, which is the shadow of R.

O is dropped vertically in o. The ground intersection is drawn parallel to the P.P.. The ray is drawn from O at an angle of 30° to the G.P. meeting the ground intersection in 3 which is the shadow of O. Join 3 I, it is the shadow of R O, the arm of the cross.

C is dropped vertically in c. The ground intersection is drawn parallel to the P.P..

The ray is drawn from C at an angle of 30° to the G.P. meeting the ground intersection in 2, which is the shadow of C. Join 2 c, it is the shadow of C c.

EXERCISE 79. Place the same cross with the arm RO vanishing to the left at an angle of 45° with the Picture Plane. Let the sun's rays come from the right, the other conditions being as in Problem 38.

PROBLEM 39, Fig. II. *To project the shadows of a square slab and a vertical rod when the sun's rays are parallel to the Picture Plane, the sun being on the spectator's right. The rays are inclined at angles of* 30° *with the Ground Plane.*

THE slab is shown at ABC abc, and the rod at Rr.

For the rod. Drop R to the ground in r, draw the ground intersection parallel to the P.P. It meets the vertical surface of the slab in X.

The vertical plane containing the ray and passing through the rod, meets the vertical surface of the slab in X. The two vertical planes will have a vertical intersection.

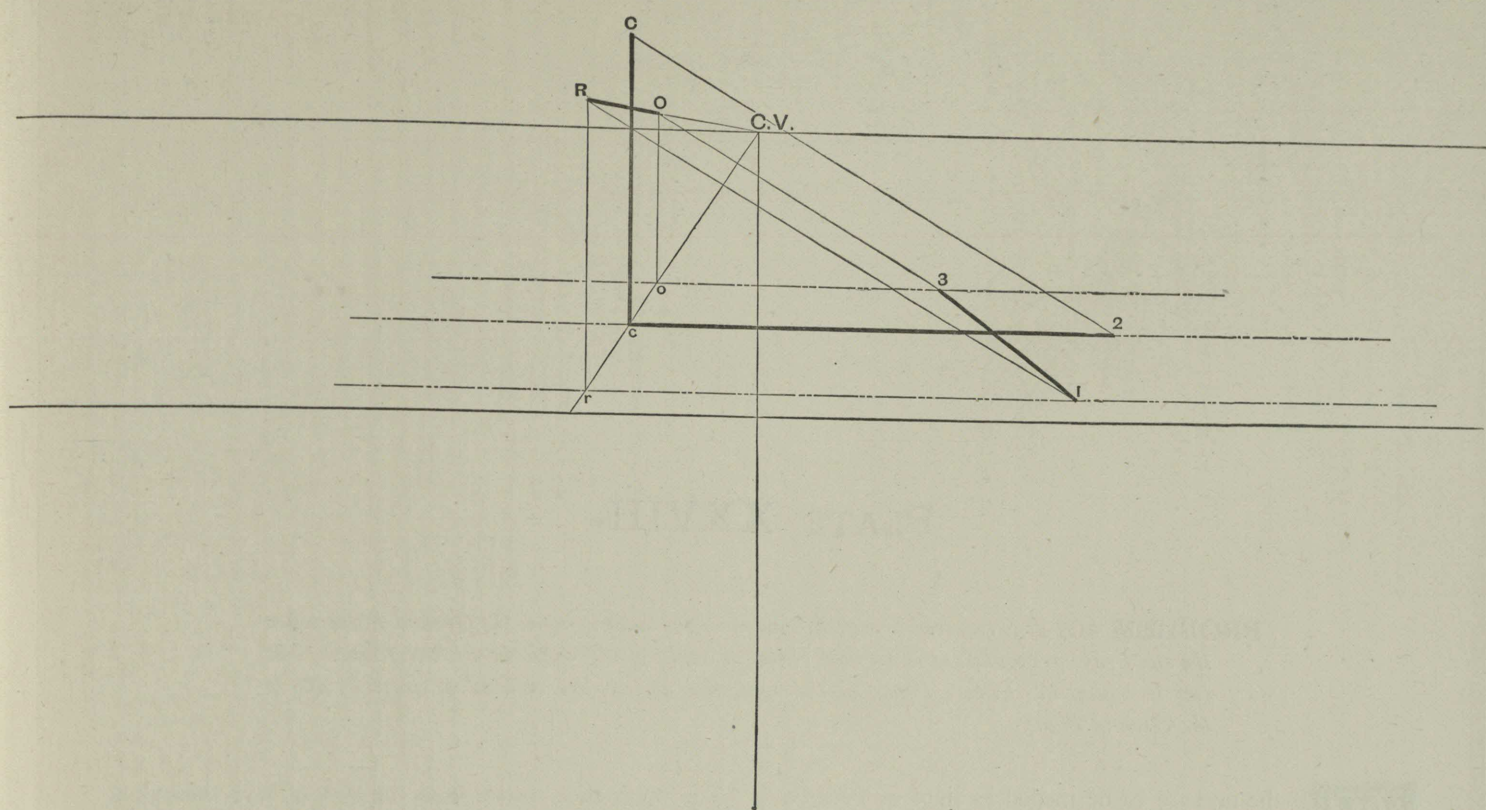
Draw a vertical line XZ to represent the vertical intersection.

From Z draw the horizontal intersection *with the upper face of the slab* of the vertical plane containing the ray in ZV.

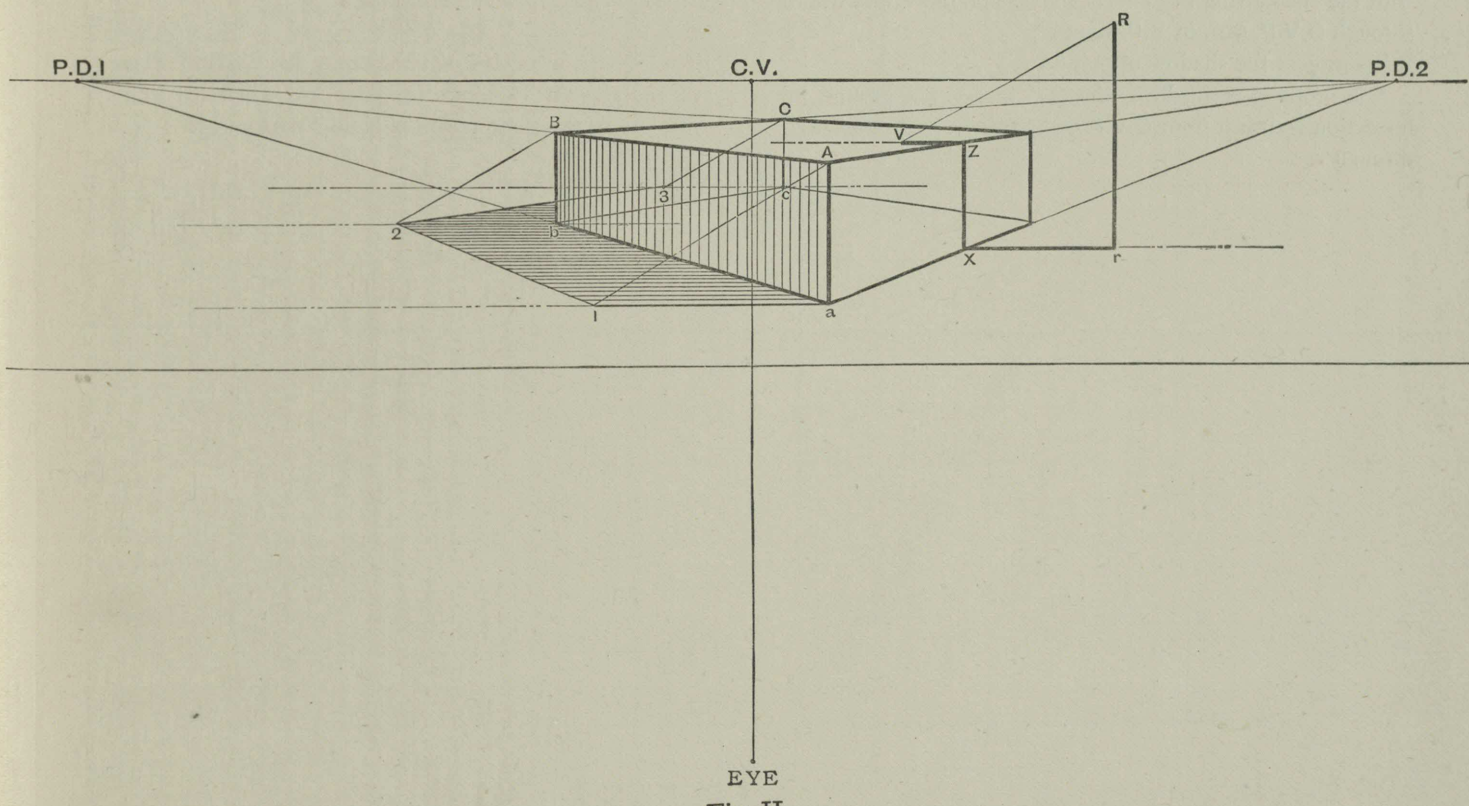
Draw the ray from R to meet the horizontal intersection in V. RXZV is the shadow of the rod upon the G.P. and the slab.

The shadow of the slab upon the G.P. will be easily effected by reference to the preceding problems.

EXERCISE 80. Cast the shadow of the cube in Problem 8, page 18, upon the Ground Plane when the light is 6 to the left, 3′ 6″ within the picture, and 15′ above the Ground Plane.



EYE
Fig.I.



EYE
Fig. II.

PLATE XXVIII.

PROBLEM 40. *It is required to cast the shadow of the building upon the Ground Plane when the sun's rays are contained in vertical planes at angles of* 60° *with the Picture Plane. The sun is behind the Picture Plane, on the right, and its rays are inclined at angles of* 50° *to the Ground Plane.*

AS the rays are to be treated as lines in Perspective we must find the V.P. of the rays, by first drawing *Van. Parallel of Vertical Planes containing rays,* at an angle of *60°* with the *Directing Line,* giving O.V.P. 60°. Through O.V.P. 60° is drawn the V.V.L. of the vertical planes containing the rays.

From O.M.P. 60° draw *A. V. Parallel of rays at* an angle of *50°* with the H.L. giving the V.P. of the rays on the V.V.L. drawn through O.V.P. 60°, in V.P. Rays 50°.

To project the shadow of point A.

A is dropped vertically to the G.P. in a. The ground intersection is drawn from O.V.P. 60° *of planes containing rays,* through a.

The ray is then drawn from the V.P. of rays through A, meeting the ground intersection in 1, which is the shadow of A.

Find the shadows of all the other points by repeating the same process, *i.e.* dropping each point vertically to the G.P. drawing the chain intersection from O.V.P. 60° of planes containing *rays,* and the ray from the V.P. of rays through the point to meet the ground intersection.

EXERCISE 81. Cast the shadow upon the Ground Plane of the slab in Problem 10, page 20, when the sun's rays are in the same position as for Problem 40.

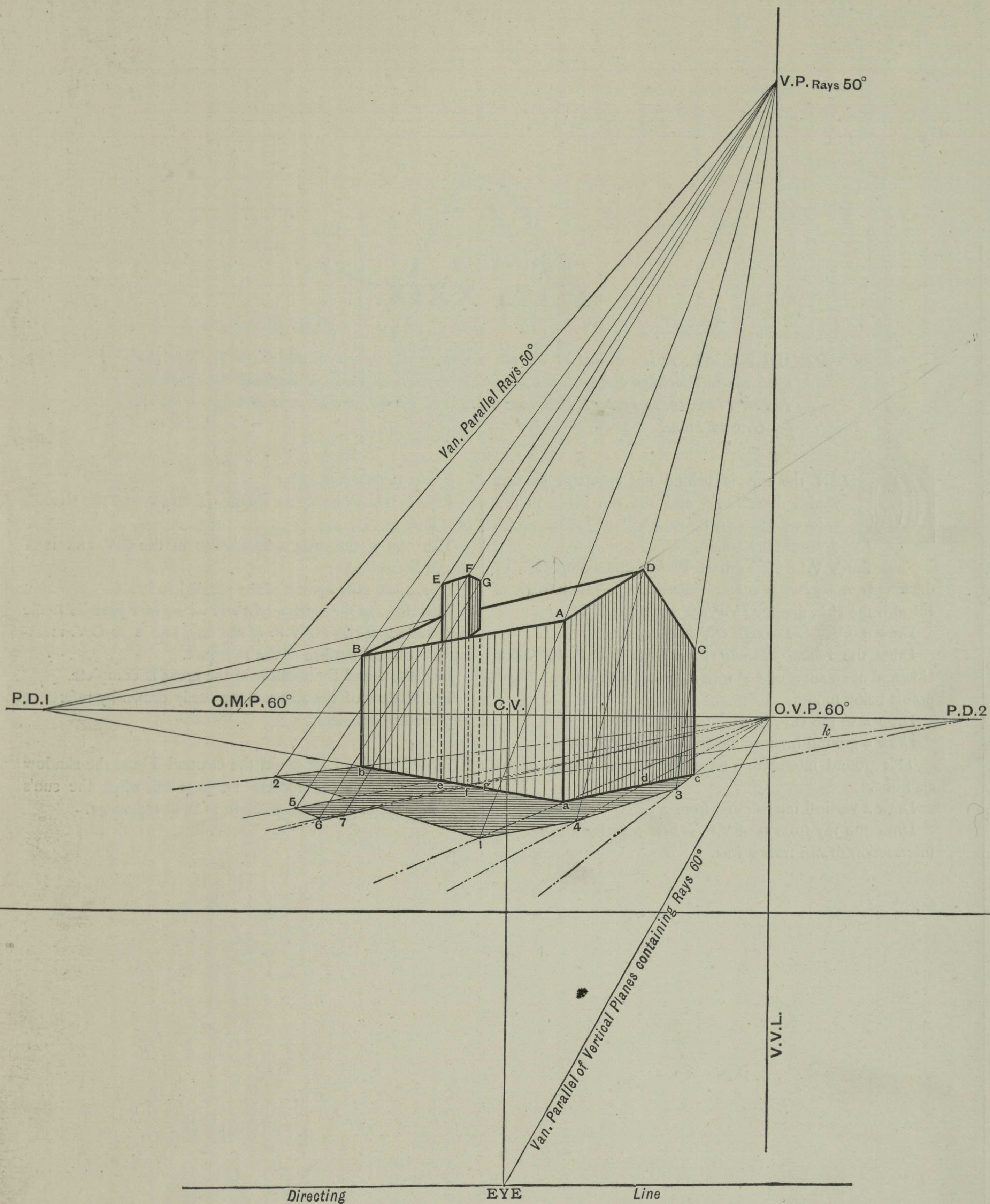

V.P. Rays 50°
Van. Parallel Rays 50°
E
F
G
D
A
B
C
P.D.1
O.M.P. 60°
C.V.
O.V.P. 60°
P.D.2
k
b
2
e
f
g
d
3
c
a
5
6
7
1
4
Van. Parallel of Vertical Planes containing Rays 60°
V.V.L.
Directing
EYE
Line

PLATE XXIX.

PROBLEM 41. *It is required to cast the shadow of the cross upon the Ground Plane and upon the vertical slab, when the sun's rays are in vertical planes at angles of* 40° *with the Picture Plane, the sun being behind the spectator, on the left, and the rays at angles of* 45° *to the Ground Plane.*

WHEN the sun is behind the spectator its rays vanish downwards, and the sun being in this case on the left, its rays will of course vanish downwards *to the right.*

Find the **V.V.L.** of Vertical Planes containing rays by drawing *Vanishing Parallel of Planes containing rays*—at an angle of *40°* with the **H.L.** giving **O.V.P. 40°** upon the **H.L.** .

Draw the **V.V.L.** through **O.V.P. 40°**.

Draw the *Vanishing Parallel of rays* from **O.M.P. 40°** *below* **H.L.** and at an angle of **45°** with it, giving **V.P. of Rays.** These points being found,

Drop **R** to the **G.P.** in **r**.

Draw a ground intersection from **r** to **O.V.P. 40°**.

This ground intersection meets the vertical surface of the slab in **2**.

Draw a vertical intersection from **2**.

Draw the ray from **R** to **V.P. of Rays** meeting the vertical intersection drawn from **2** in **4**.

4 2 r is the shadow of **R r**.

Find in the same way the shadow of **o** in **3**, and the shadow of **c** in **5**.

The arm of the cross **c o** vanishes in the **C.V.** and is of course horizontal.

Its shadow falls upon a plane parallel to itself.

Therefore, as *the shadow of a line on a plane parallel to the line, will vanish to the* **V.P.** *of the line*, join **5** to **C.V.** meeting the vertical surface of the slab in **6**.

Join **6** to **3** and the shadow of the cross is complete.

The shadow of the slab will be easily drawn by reference to preceding problems.

EXERCISE 82. Cast upon the Ground Plane the shadow of the steps in Problem 14, page 26, when the sun's rays are in the same position as in Problem 41.

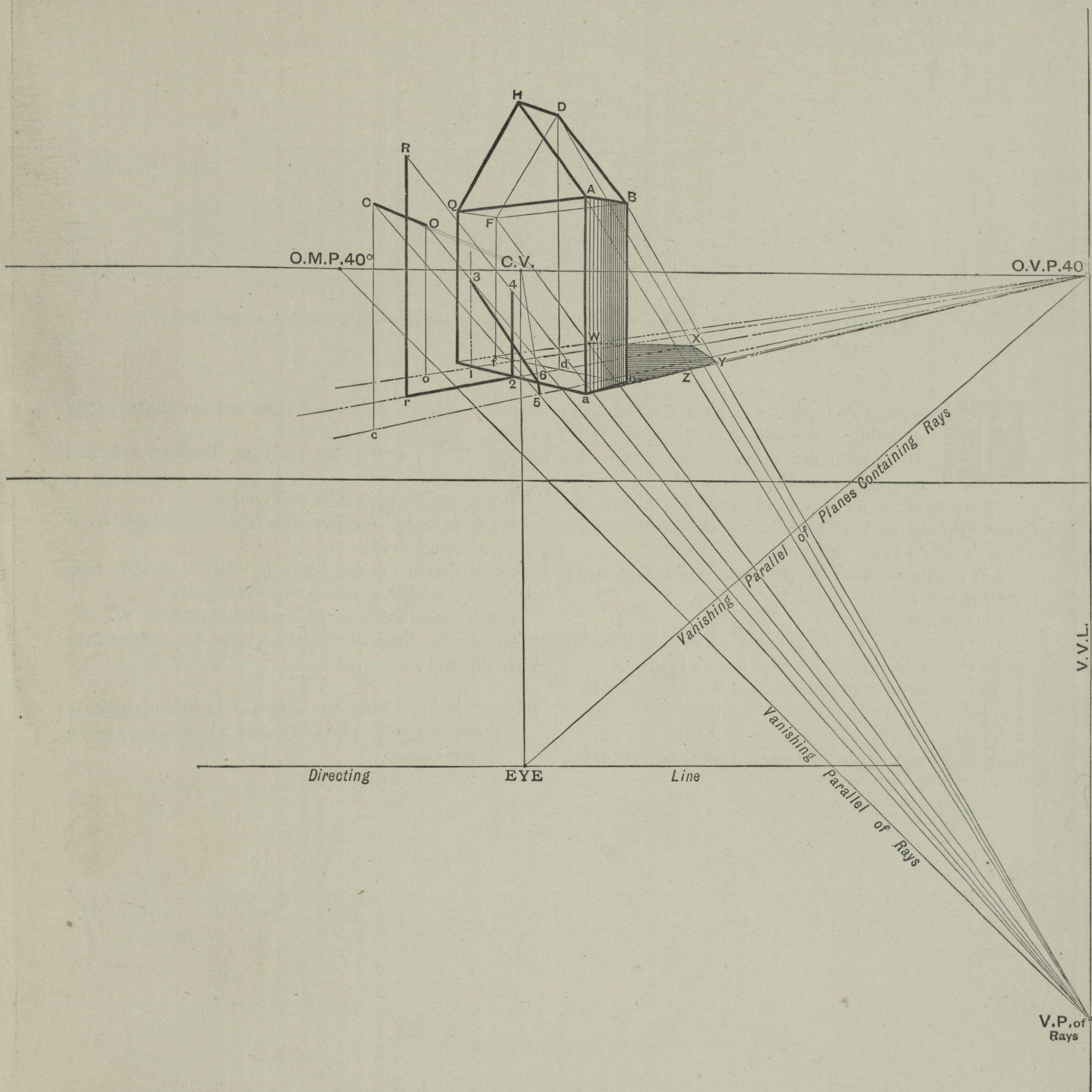

H
D
R
A
B
C
Q
O
F
O.M.P.40°
C.V.
O.V.P.40°
3
4
W
X
f
d
Y
o
i
2
6
b
Z
r
5
a
c
Vanishing Parallel of Planes Containing Rays
V.V.L.
Vanishing Parallel of Rays
Directing
EYE
Line
V.P. of Rays

PLATE XXX.

PROBLEM 42. *It is required to find the reflection of the figure* X, Y, W, Z, P, T, *in the still water represented in* **Fig. I.**

IF we find the reflection of point X, X is dropped vertically to the surface of the water in P.. The vertical line X P is continued, as it were *underneath* the surface of the water.

Then measuring from P to X and transferring this measurement upon the vertical line *below* the surface of the water in x, we have x as the reflection of X.

x is exactly the same distance *below* the surface of the water as is X *above* the surface.

x is *vertically below* X.

The reflection of W will be found by measuring the distance P W from P, upon the vertical line P x in w.

This is the rule for all reflections in still water. Drop the point *vertically* to the *surface* of the water. Produce the vertical line *below* the surface of the water. Measure from the surface, upon the vertical line below, the actual distance between the original point and the surface of the water.

The figure A B C H K L on the right illustrates the same principle.

In **Fig. II.** the same rules are exemplified.

For example, the nearest surface of the projecting piece of wood is parallel to the P.P..

Draw through the two points in which it emerges from the water, a *ground*, or rather, *water intersection.*

Drop points P and O to this intersection in 2 and 3.

Measure the distances 2 P and 3 O below the surface of the water in p and o.

EXERCISE 83. Find the reflection of the pyramid in Problem 9, page 18, the Ground Plane being the reflecting surface.

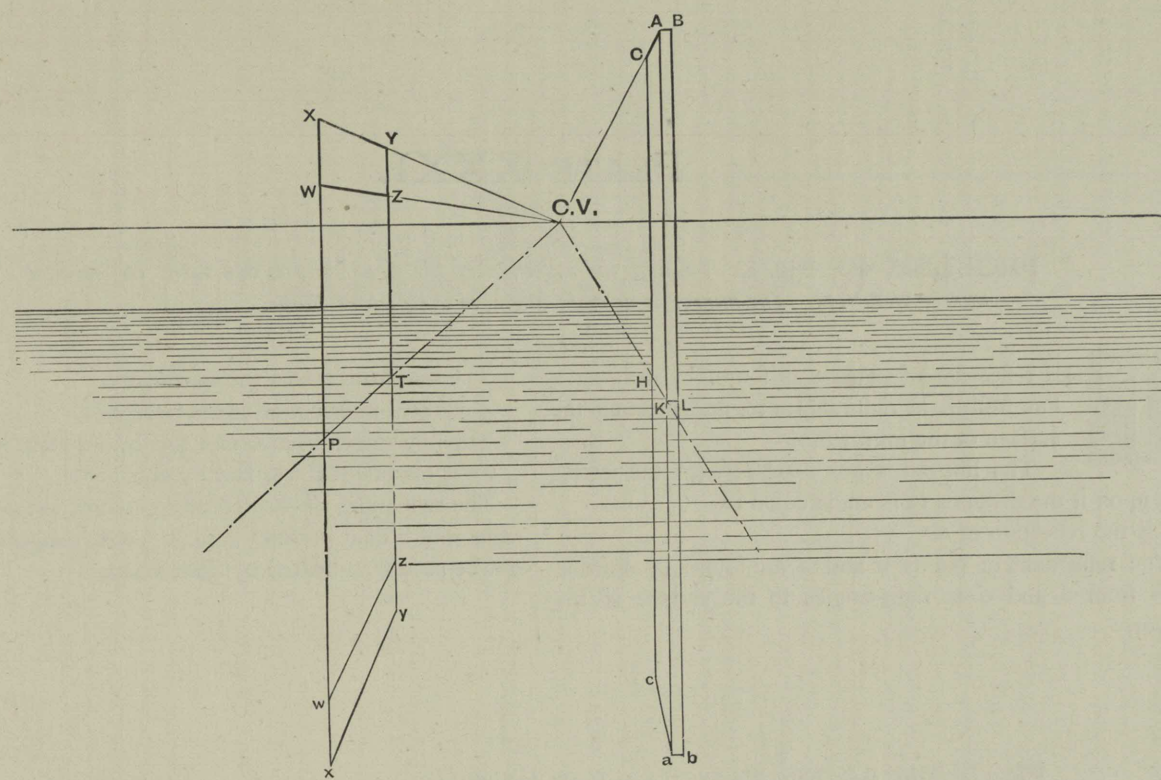


Fig. I.

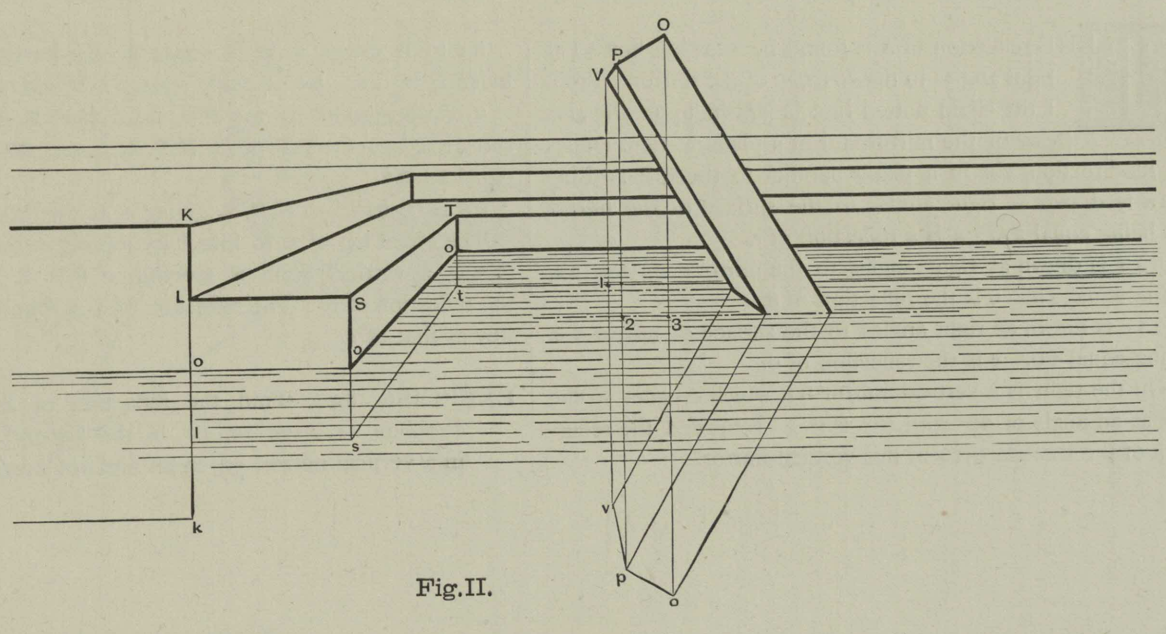


Fig. II.

PLATE XXXI.

PROBLEM 43, Fig. I. *To find the reflection of the figure* A, B, C, D, E, F, *in the vertical mirror* W, X, Y, Z. *The mirror is at right angles to the Picture Plane.*

THE reflection of point A is found by drawing a line from A, at right angles to, and meeting the surface of the mirror in I.

The line A I is produced *through* the mirror and upon it the distance I a is made equal I A.

a is the reflection of A.

The reflections of points B and C are found by drawing lines from B and C at right angles to the surface of the mirror.

The line from C cuts the mirror in 3.

A vertical intersection is drawn from 3.

The line drawn from B cuts this intersection in 2.

2 b is made equal 2 B, and 3 c equal 3 C.

The reflections of the remaining points are found in the same way. That portion of the reflected image which would not be seen is indicated by dotted lines.

PROBLEM 44, Fig. II. W, X, Y, Z, *is an inclined mirror.* A, B, C, *is a vertical triangle. Required the reflection of the triangle in the mirror. The mirror is at right angles to the Picture Plane. The triangle is parallel to the Picture Plane.*

THE reflection of A is found by drawing line A I at right angles to the *direction* of the mirror. From I the chain dotted line is drawn upon the *surface* of the mirror and in a plane at right angles to its direction, viz. in a plane parallel to the P.P.. Line A 2 a is drawn at right angles to the surface of the mirror 2 a being equal 2 A. a is a reflection of A.

B 3 b is drawn at right angles to the mirror's surface, 3 b, being equal 3 B. b is the reflection of B.

C 4 c is drawn at right angles to the surface of mirror 4 c, being equal 4 C. c is the reflection of C.

On the right is a vertical mirror H K which recedes to the left at an angle of 45° with the P.P.. Required the reflection of the triangle A B C in this vertical mirror.

B 5 b′, is drawn at right angles to the surface of mirror, meeting it in 5. 5 b′ is made equal 5 B by means of M.P. 2.

B is transferred to the P.P. from M.P. 2. in B 2. 5 is transferred to the P.P. from M.P. 2. in 52. 52, B R, is made equal 5 2, B 2.

B R is returned to M.P. 2, giving b′ as the reflection of B.

The reflection of C is found by joining C to P.D. 2, and raising a vertical from b′ meeting C P.D. 2, in c′. c′ is the reflection of C. The reflection of A is found in the same way as that of B.

EXERCISE 84. Find the reflection of the prism in Problem 11, page 20, 1st, in the Ground Plane, 2nd, in a vertical mirror, 3d, in an inclined mirror.

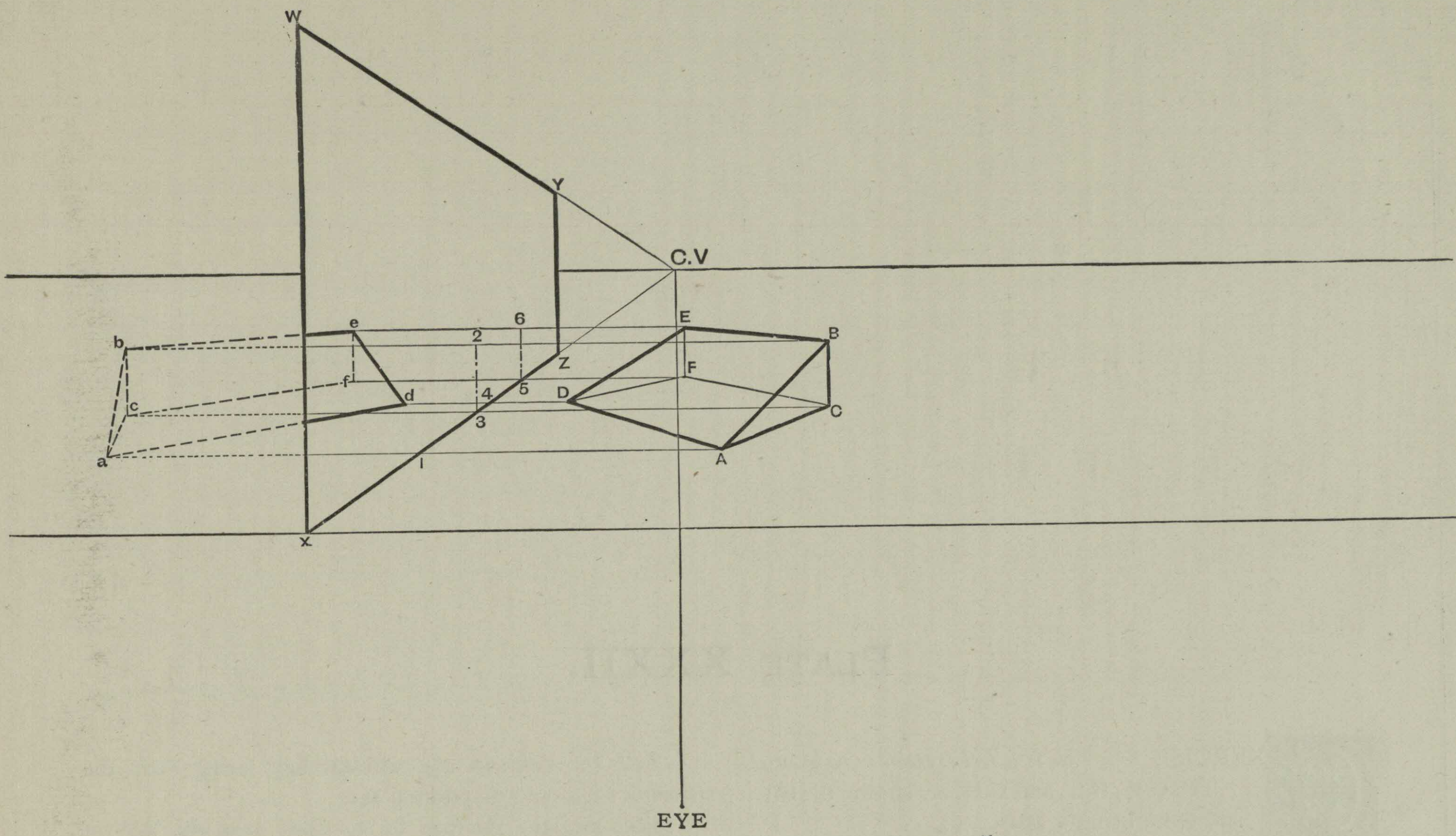

W
Y
C.V
b
e
2
6
E
B
Z
F
f
c
d
4
5
D
C
3
a
I
A
X
EYE

Fig.I.

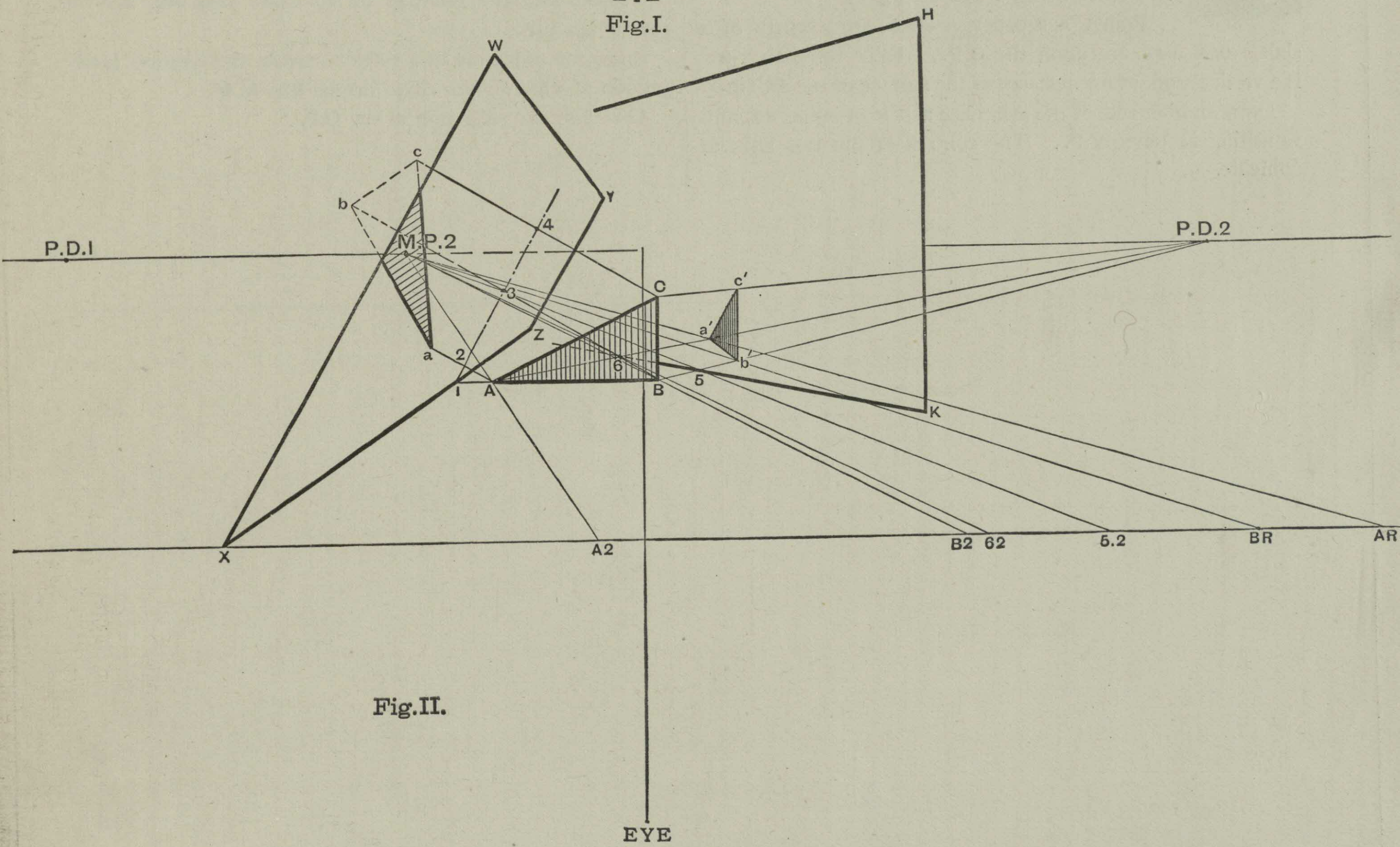

H
W
c
b
Y
4
P.D.I
M.P.2
P.D.2
3
C
c′
a′
Z
b′
a
2
6
I
A
B
5
K
X
A2
B2
62
5.2
BR
AR
EYE

Fig.II.

Plate XXXII.

EXERCISE 85. *This is a test exercise to be worked.* The C.V., H.L., and G.L., are shown, the H.L., and G.L., being 5′ apart.

Points A, B, C, D, E, F, and G, are corners of a slab. D, A, and E, are upon the G.P.. Draw the slab. RX is a vertical rod, which rests upon the surface of the slab in X.

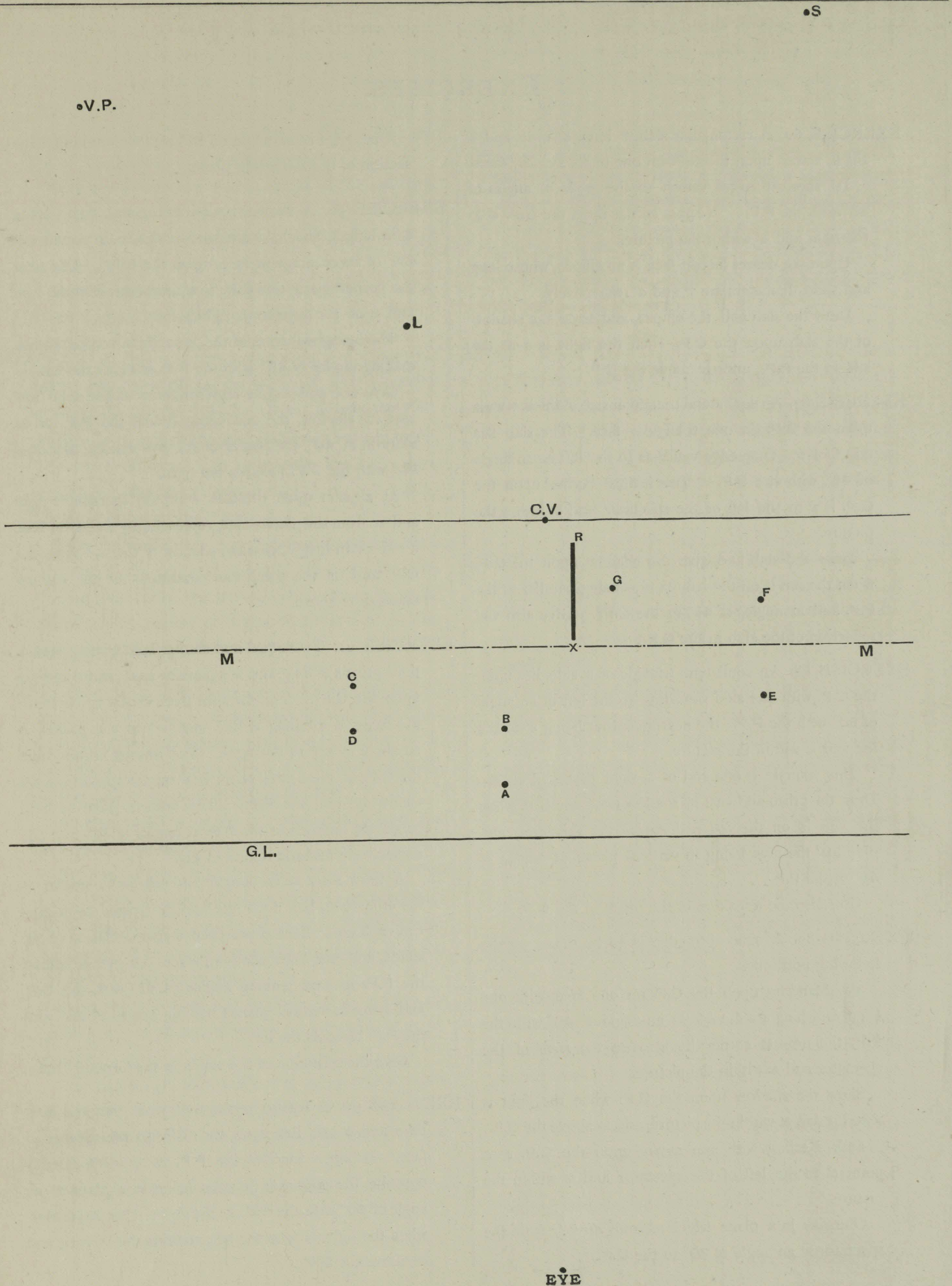
Draw another rod of the same size as RX, crossing RX, and vanishing in point V.P.. The centres of the two rods to coincide.

Project the shadows, the artificial light being 4′ on the spectator's left, and its position at L.

Project also the shadows by the sun's rays, the V.P. of rays being at S.

Show the reflection in a vertical mirror, the Ground Intersection of which is the chain dotted line M M.

Give also the reflection in the G.P..


S
V.P.
L
C.V.
R
G
F
M
X
M
C
E
B
D
A
G.L.
EYE

Exercises.

EXERCISE 86. A rectangular slab, 7′ long, 5′ wide, and 3′ thick, stands upon the G.P. on one of its *smallest* faces.

Its shortest edges vanish to the *right* at angles of 45° with the P.P.. Nearest corner upon the G.P. 4′ to the right and 5′ within the picture.

Upon the nearer largest face is an ellipse, whose long and short diameters are 7′ and 5′ respectively.

Draw the slab and the ellipse, and show the shadow of the slab upon the G.P. when the light is 3′ to the left, in the P.P., and 20′ above the G.P..

EXERCISE 87. An equilateral triangular slab 3′ thick, stands upon the G.P. on one triangular face. The slab has edges of 9′. One edge vanishes to the left at an angle of 45° with the P.P.. The nearest corner upon the G.P. is 2′ to the left of the spectator, and 5′ within the picture.

Draw the slab and give the shadow upon the G.P. when the sun is on the left, its rays being parallel to the P.P. and at angles of 45° to the G.P.. Give also the reflection of the slab in the G.P..

EXERCISE 88. An equilateral triangle of 6′ side, lies upon the G.P. with one side vanishing to the left at an angle of 50° with the P.P., the nearest corner being 3′ to the *left* and 4′ within the picture.

This triangle is one end of a right prism, of 4′ axis. Draw the prism and cast its shadow upon the G.P. when the sun is on the *left* and *behind* the spectator, altitude 30° the rays being in vertical planes at angles of 45° to the P.P..

Give also the reflection of the prism in the G.P..

EXERCISE 89. A cube of 7′ edge is to be drawn in the following positions.

1st. Standing upon the G.P. on one face, with one edge vanishing to the *left* at an angle of 40° with the P.P. the nearest corner being 3′ to the *right* of the spectator and 4′ within the picture.

Show the shadow upon the G.P. when the light is 3′ to the *left*, 3′ within the picture, and 18′ above the G.P..

2nd. Resting with one *corner* upon the G.P. at a point 5′ to the left of the spectator and 6′ within the picture.

One *face* in a plane which ascends *directly* from the spectator at an angle of 30° to the G.P..

One edge at an angle of 35° with the Ground Intersection of the ascending plane.

EXERCISE 90. A short, regular, hexagonal, right prism, 3′ in length, and having a base of 4′ side, is placed with one of its rectangular faces upon the G.P.. The nearest corner upon the G.P. is 9′ from the Picture Line and 2′ on the spectator's right hand.

The hexagonal faces of the prism lie in vertical planes, making angles of 40° with the P.P. towards the left.

Give the perspective representation of the solid and cast its shadow, the sun being *behind* the P.P. at an altitude of 40°, the planes of its rays making angles of 70° with the P.P. towards the right.

At a point upon the G.P. 8′ on the spectator's right and 21′ from the picture line, represent, by a vertical line, a rod 20′ in height, cast the shadow of the rod upon the G.P. and on the solid, the conditions of the sun remaining as above.

EXERCISE 91. Two equal and similar solids being square right prisms 9′ long and 4′ square at base, stand upright upon the G.P.. The distance from centre to centre of the bases is 10′ along a line inclined to the picture at an angle of 40° to the right. The nearest corner upon the G.P. of the nearer prism is 4′ on the spectator's left and 12′ beyond the P.P.. The vertical faces of both solids are inclined to the P.P. at angles of 40° and 50° respectively towards right and left.

A third solid 16′ in length but otherwise similar to the other two, lies upon the upright prisms projecting 1′ at each end. Give a perspective representation of the solids, and cast their shadows when the sun is behind the P.P. its rays making angles of 45° with the G.P. and lying in vertical planes making angles of 30° with the P.P. towards the left.

Give the reflection of the solids in the Ground Plane.

EXERCISE 92. A regular pentagonal right pyramid of 5′ base and 6′ axis lies upon the G.P. on one triangular face. Its apex touches the P.P. at a point directly opposite the spectator, its axis being in a plane at an angle of 40° with the P.P. to the right. Cast its shadow when the light is 5′ to the left, 1′ within the Picture, and 14′ above the G.P..

EXERCISE 93. A disc (or portion of a right cylinder) 3′ in length, and 8′ in diameter rests upon the G.P.. Its bases are in vertical planes, and inclined towards the right at angles of 45° with the P.P., and the nearest point of the line of its contact with the G.P. is 3′ on the left of the spectator and 8′ from the P.P..

Give the perspective representation of the solid and its shadow when the sun is behind the P.P. at an altitude of 50°, and in a vertical plane inclined to the left at an angle of 30° with the picture.

Show the reflection of the disc in a vertical mirror inclined at 45° with the Picture Plane.

EXERCISE 94. Show in perspective a block of 5 steps, the lowest step being nearest the spectator, with its nearest angle on the ground 7′ to the left and 3′ from the P.P.. Each step is 6″ high (6″ riser) and 12″ broad (12″ tread). The block of steps is to be 8′ wide, the long edges, which are horizontal, vanishing towards the left, making angles of 40° with the P.P..

Show the shadow on the ground plane of the block of steps in the required position, supposing the sun's rays to come from the left of the spectator, to be inclined to the ground at angles of 50° and to be contained by planes parallel to the P.P..

EXERCISE 95. A right prism having for its section a triangle of 6′ base and 4½′ altitude, lies with its largest side upon the G.P.. Its nearest angle is 6′ to the left and 4′ from the P.P.. The axis, which is 6′ in length, vanishes to the right at an angle of 50° to the P.P..

Another prism 4′ square in section, axis 8′ long and in a vertical position, stands upon the ground in front of the nearest side of the first one; its axis and two diagonals being in a vertical plane which is perpendicular to the axis of the triangular prism, and passes through its centre. The axis of the square prism is 7′ from the centre of the triangular one.

Show the whole of the shadow cast by the square prism upon the triangular one or the ground as the case may be.

The rays of light to be inclined to the ground at angles of 40° and lying in planes parallel to the one which contains the centre of the first prism and the axis of the second.

Show also the reflection of the solids in the Ground Plane.

EXERCISE 96. Arrange three rods of 6′, 9′, and 10′ high respectively, so that their shadows upon the G.P. by different lights shall be equal in length and meet in the same point.

EXERCISE 97. A vertical line 7′ long, 10′ within the picture and 6′ to the right of spectator throws a shadow upon the G.P., the perspective representation of which is a line 1½″ long inclined towards the left at an angle of 25° to the ground line of the picture; find the inclination of the ray casting the shadow and the angle made with the P.P. by the plane containing it.

EXERCISE 98. Erect a vertical line 7′ high at a point 4′ to the right, and 8′ within the picture, and shew the shadow which would be cast in four different directions by rays of light proceeding from before and behind, on the right and the left of the spectator; in every case the ray to be inclined to the ground and in a plane making an angle of 45° with the picture.

EXERCISE 99. A rectangular solid 6′ wide, 4′ thick and 12′ long lies on the Ground Plane, its longest edges inclined to the P.P. at angles of 55° and vanishing to the right. Its nearest angle is 6′ to the left, and 6′ beyond the picture.

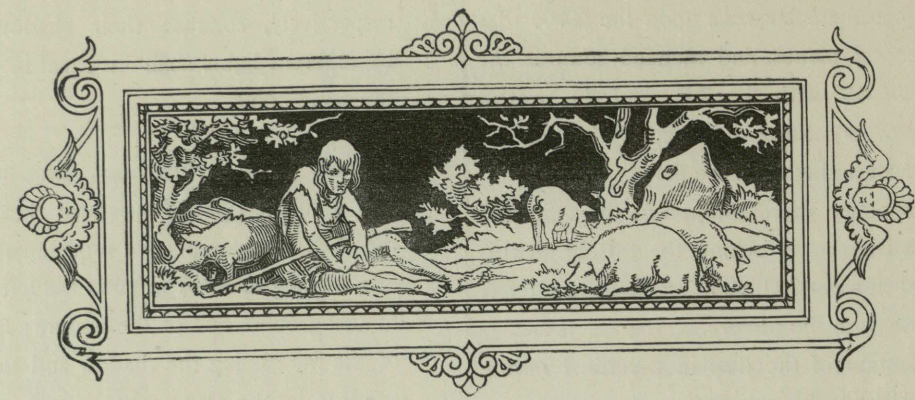
Behind it stands a pyramid with axis 8′ 6″ high and vertical, its base a regular pentagon of 4′ side, one of its angles touching the centre of the lower edge of the solid furthest from the P.P., and a diagonal vanishing with the long edges of the solid at 55°.

The shadows of the two blocks are to be thrown towards the spectator from left to right, by rays inclined at angles of 45° to the G.P. and contained in vertical planes making angles of 50° with the P.P..

Show the reflection of the solids in a vertical mirror parallel to the Picture Plane.

EXERCISE 100. A cube (3′ edge situated at pleasure near the centre line of the picture and 2′ within it,) has its shadow cast upon the G.P. by rays falling upon the object from behind the spectator, over his right or left shoulder.

Any regular prism with square base situated anywhere to the right of the spectator, and within the picture, has its shadow determined upon the G.P. by rays proceeding from behind the object.



THEORY OF SHADOWS AND REFLECTIONS.

IN investigating the theory of Shadows and Reflections it will be necessary for us to consider several facts relating to the nature of Light.

Light diminishes in intensity as it recedes from the point of emission.

Various natural substances, such as iron, wood, brick, &c., prevent the passage of light, and are hence called *opaque.* Other substances, such as glass, crystal, &c., allow light to pass freely through them, and are termed *transparent;* while some permit only a partial passage of light, and are named *translucent.*

When light falls upon a polished surface, such as the surface of a mirror, it is *reflected* from that surface. If the surface is a plane, the light will be reflected at angles which will bear certain relations to the angles at which the light falls upon the surface.

If we exclude all the sun-light from a room, excepting what may pass through a small hole in one wall, we shall be able

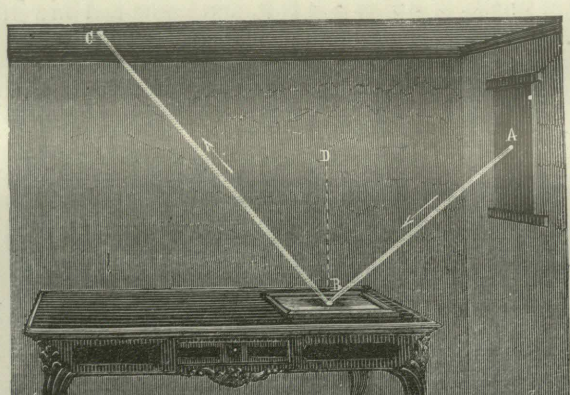


Fig. 28.

to see a distinct ray of light travelling in a *downward* rectilinear direction (see A B Fig. 28). If we place a plane mirror in the path of the ray, we see that its course is changed, the ray now proceeding *upwards* in the direction B C. This is the process of reflection, B C being the reflected ray. The *real* ray A B is called the *incident* ray. If we imagine a vertical line B D to proceed from B, we shall be enabled to compare very easily the directions of the two rays, and we find the two angles, viz. angle A B D and angle D B C, are *equal.*

The angle A B D is called the *angle of incidence*, and the angle D B C, the *angle of reflection.* Thus the angle of reflection is equal to the angle of incidence, and when the surface of the mirror is in a horizontal position the two rays are contained *in the same vertical plane.*

This may be proved by making use of a contrivance similar to that shown in Fig. 29.

A small plane mirror is placed horizontally in the centre

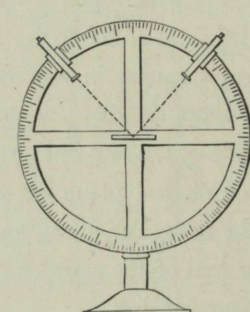


Fig. 29.

of a vertical circle. The circle is divided into a number of equal parts for the purpose of measurement. Two small movable tubes are fixed on the circumference of the circle. If a candle is placed so that its light shines through one of the tubes upon the mirror in the centre, we find, on moving the other tube to a position in which it receives the reflected ray, that both tubes are at the same distance from the highest marked division upon the circumference of the circle. Thus the angles of reflection and incidence will be proved to be equal.

It may be also proved by taking a tube bent as in **Fig. 30**, and placing its lower end, *which must be open*, upon the surface of a horizontal mirror as A B C D. Then allowing the light of a candle to enter one end of the tube, place the eye at the

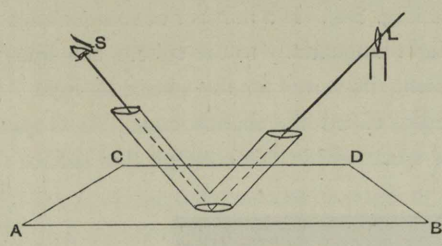


Fig. 30.

other end, and on looking in the tube the flame of the candle will be seen, *if the two tubes are at the same inclination* with the surface of the mirror. If the inclination of one tube is not equal to that of the other, the light will be no longer seen. It should also be borne in mind that the tubes must be so placed that *a vertical plane* would pass through their centres.

To make this more clear let us refer to **Fig. 31.**

M M represents the surface of a mirror, L a light, and S the spectator's eye. The incidental ray proceeds from the light, meeting the mirror in R. The reflected ray proceeds from

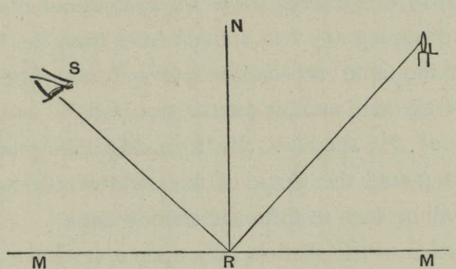


Fig. 31.

the mirror at R, and passes to the eye at S. Angle L R M is equal to S R M, and if we imagine a line N R (called the *normal*), at right angles to the surface of the mirror, angle L R N is equal to angle S R N, or in other words, the *angle of reflection* is equal to the *angle of incidence.*

Let us now see how this is related to the reflected appearance of any object in a polished surface. If we hold a candle in front of a mirror we see the reflected image of the candle at a certain distance from us. If we place the candle near to the mirror the reflected image is near to us. If we move the candle away from the mirror the image appears to recede. We thus see that the *position* of an object in relation to the reflecting surface, influences the position of its reflected image.

In **Fig. 32,** M M represents a mirror, L a light, and **S** the spectator's eye.

L F, and L R, are two incidental rays proceeding from L and falling upon the mirror at F and R respectively.

F X and R Z are the two corresponding reflected rays, rising from the mirror to the spectator's eye.

If we imagine the two reflected rays to be produced

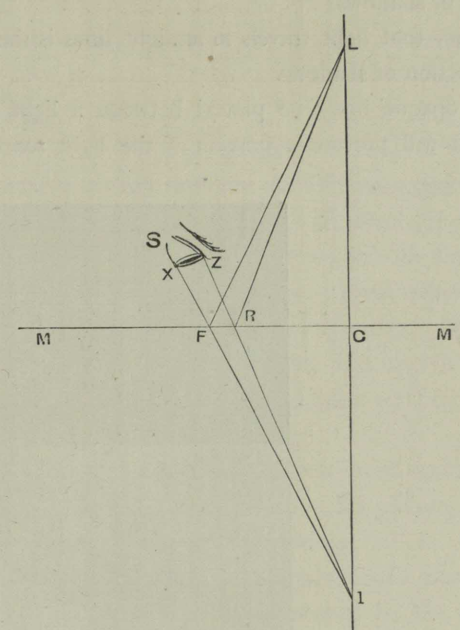


Fig. 32.

through the surface of the mirror they would meet at l, which is the same distance from the surface of the mirror, below it, as the light L is above it.

Therefore the eye will have the impression that the rays proceed from l, so that the image of the light appears to be as much below the mirror as the light is situated above its surface.

In **Fig. 33** we have the side view of a plane mirror, a

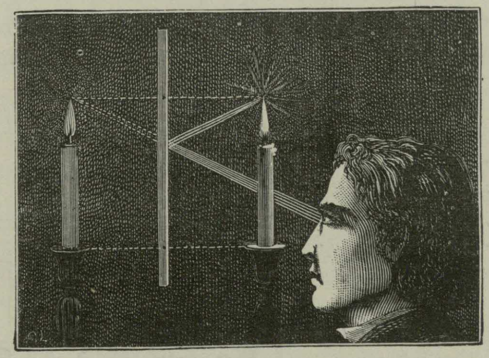


Fig. 33.

candle, its reflection, and the person observing it. It will be easily seen by means of the represented rays and by reference to **Fig. 32,** that the reflected image of the candle flame is the same distance from the surface of the mirror behind, as the flame itself is in front.

A careful study of Problems 42, 43, and 44 will, on comparison with the foregoing explanation, render the study of reflections in plane surfaces quite clear. The study of reflections in spherical and conical surfaces being quite out of the scope of an elementary work, we will proceed to the question of shadows.

The fact that light travels in straight lines is the basis of the projection of shadows.

If an opaque body be placed between a light and any surface, it will prevent a portion of the light reaching the surface. The shape of the intercepted object will influence the shape of the dark figure formed upon the surface by the deprivation of light.

Let us imagine the opaque intercepted body to be a spherical object such as a ball, and the source of light a candle (see **Fig. 34**). If a number of straight lines are drawn from the light tangentially to the sphere, the lines will form a cone, having its vertex in the source of light. The cone thus formed is called the shadow-cone. It is quite obvious that all the space within this cone on that side of the sphere

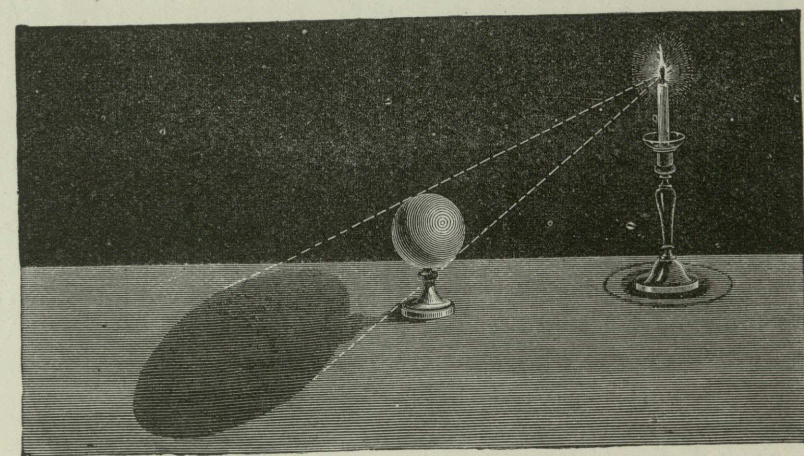


Fig. 34.

which is turned away from the light, is completely hidden from the rays. The intersection of the shadow-cone with the surface receiving it defines the shape of the shadow.

The actual method of projecting the shadow is effected by imagining the ray required to determine the shadow of any point in an object, to be contained in a vertical plane. The intersection of this vertical plane with the plane of shadow is first determined, and the ray drawn from the source of light through the point in the object to meet the intersection (see page 74).

For example, the shadow of the sphere could be fixed upon the table in the following manner. Take any point upon the sphere to which a *ray from the light would be a tangent.* Find (by dropping any two vertical lines from the ray to the surface of the table) its shadow intersection. The point of contact of ray and shadow intersection will be one point in the limit of the shadow. By repeating this process in a number of points the shape of the shadow will result, and the rays will be seen to form the shadow-cone.

If a portion of the shadow falls upon a vertical surface, instead of a *horizontal* shadow intersection we shall have a *vertical* one. In like manner if a portion of shadow falls upon an inclined surface, we shall have an inclined shadow intersection.

APPENDIX.

If we have carefully worked through all the Problems contained in this work (with the exception of Problems 18, 19, 20, 22, and 23, which are examples of Second Grade Examination Papers of the Science and Art Department, and are only inserted for the benefit of those wishing to sit for the Department Examinations), and carefully considered the theoretical illustrations, we shall have a fair knowledge of the Science of Perspective.

The next object should be to know how this scientific knowledge may be practically applied to the actual work of drawing from nature.

A complete consideration of this subject would of course be rather beyond the aim of this little work, but a few points may be suggested to the student.

The first is in regard to the distance of the spectator from the object he is supposed to draw.

This distance has been given in most of the Problems we have worked, as twelve feet; but this is only adopted as a convenient distance for the working of Perspective problems.

The real distance chosen in actual drawing depends upon the size of the object to be drawn, and it may be safely taken as a rule that the spectator should be at a distance from the object equal to about three times its greatest dimension.

The height of the spectator's eye from the ground of the picture is another point for consideration. This also depends upon the size and position of the objects to be drawn. For example, if we are drawing the cups and saucers upon a tea-table, the table forming the ground of our picture, the eye of the spectator may be but a few inches from the ground of the picture. In like manner, if the spectator is upon a mountain, drawing the scenery below, the eye will probably be at a distance of many yards from the ground of the picture.

If we have observed carefully the different appearances of the objects drawn in the plates which illustrate our problems, we shall have found that the angle made with the Picture Plane by one portion of an object has a great influence upon its appearance, whether pleasing or otherwise. Under this head it may perhaps be suggested that as a rule no two sides of an object should make equal angles with the picture, &c. &c.

It may also be remarked that the choice of position of the horizontal line, whether low or high in the picture, has great influence upon the effect of the drawing.

In drawing from nature the student will observe that reflections in water are much influenced by the degree of movement of the water. Thus, a slight ripple will have the effect of elongating and blurring the reflected image, whilst in some states of the surface the reflected image is almost entirely lost.

In regard to shadows. Our problems deal only with the source of light as a single point. This is not so in nature, all natural sources of light having length and breadth, which is the cause of the "penumbrae" usually seen upon the boundaries of shadows, the result being a great difference in the degrees of sharpness or distinctness of the *edges* of the shadows, though still following the general *shape* determined by the processes explained in our problems. As a rule, the nearer the object or portion of the object causing the shadow is to the surface receiving the shadow, the sharper and more distinct is the boundary.

The student will find much further information on this branch of the subject in the French work by Thibault and the English work of Malton, a perusal of which will well repay any one who is anxious to gain a thorough knowledge of the Science of Perspective.

INDEX.